# Using Turning Points to Solve Problems

You approach a word problem in the same way as other problems. You can use differentiation to find the maximum or minimum area or volume in the same way as we calculated turning points.

Make sure you rearrange and substitute equations so that you end with an equation with only one variable.

*Example* 1. A large cuboid tank is made from $54\text{m}^2$ of sheet metal. The tank has a horizontal base but no lid. The height is $x$ m and the vertical opposite faces are squares. Find the maximum volume.

let the length by y, the width be x and the height be x.

$$\text{volume} = x^2 y$$

$$\text{Surface area} = x^2 + x^2 + 3xy \qquad \text{also SA} = 54m^2$$

$$54 = 2x^2 + 3xy$$

$$\frac{54 - 2x^2}{3x} = y$$

substitute y into the volume formula

$$v = x^2 y$$

$$v = x^2 \left( \frac{54 - 2x^2}{3x} \right)$$

$$v = \frac{54x^2 - 2x^4}{3x}$$

$$v = 18x - \frac{2x^3}{3}$$

To find the maximum volume we differentiate

$$v = 18x - \frac{2x^3}{3}$$

$$\frac{dv}{dx} = 18 - 2x^2 \qquad \frac{dv}{dx} = 0 \text{ for a turning point}$$

$$0 = 18 - 2x^2$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3 \qquad \text{as } x \text{ is a length } x \text{ must be } 3$$

when $x = 3$ $\quad v = 18x - \frac{2x^3}{3}$

$$v = (18 \times 3) - \frac{2 \times 3^3}{3}$$

$$v = 36$$

To see if its a maximum or minimum value

$$\frac{d^2v}{dx^2} = -4x$$

when $x = 3$ $\quad \frac{d^2v}{dx^2} = -12 \qquad as \quad \frac{d^2v}{dx^2} < 0$

Volume $= 36$ is the maximum value