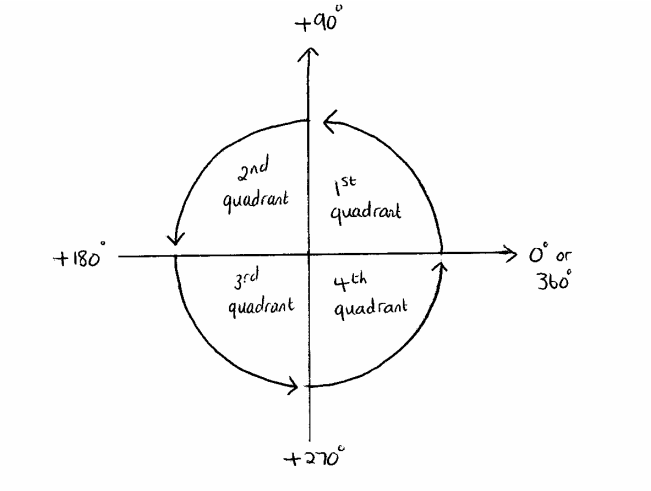
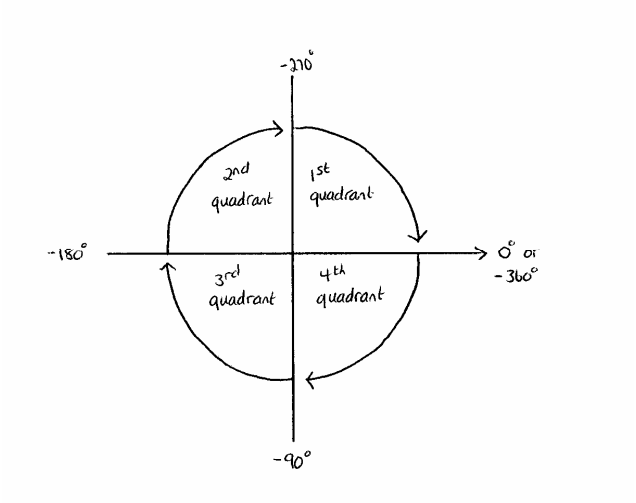
Using and Understanding Trig Functions for Positive and Negative Angles

The Three Trigonometric ratios are

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

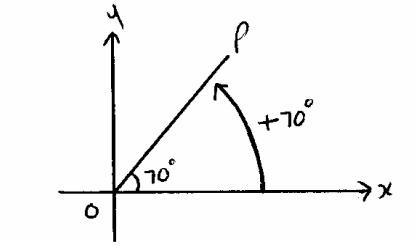
The x-y plane is divided into 4 quadrants



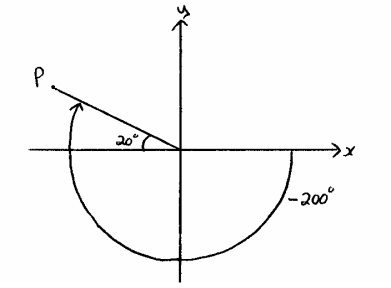


Example 1. Draw the line OP, where O is the origin, and the angle θ makes an angle with the positive x-axis.

a) +70°



b) -200°



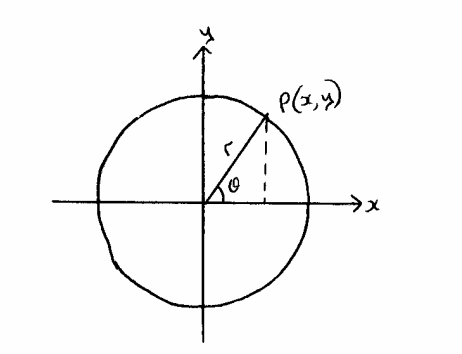
Equivalent Trigonometric Ratios

For all values of θ, the definition of sin θ, cos θ and tan θ are taken to be…

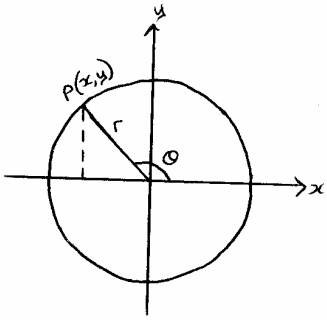
$\sin\theta = \frac{y}{r}$ where *x* and *y* are the coordinates of P and r is the length OP

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$



If θ is acute



If θ is obtuse

Some definitions are:-

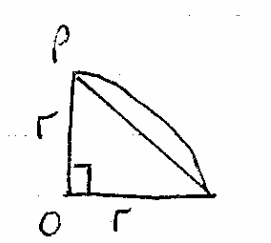
$\sin 90° = 1$	$\sin(-90°) = -1$
$\sin 180° = 0$	$\sin(-180°) = 0$
$\sin 270° = -1$	$\sin(-270°) = 1$
$\cos 180° = -1$	$\cos(-180°) = -1$
$\cos(-90°) = 0$	$\cos 90° = 0$
$\cos 450° = 0$	$\cos(-450°) = 0$

Why?

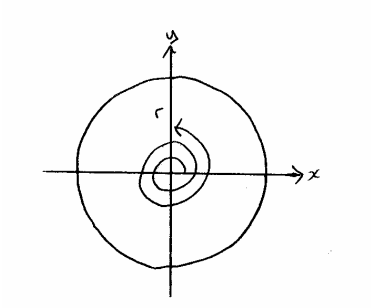
$\sin\theta = \frac{y}{r}$ for sin 90°

$\sin 90° = \frac{r}{r}$

$\sin 90° = 1$



Example 1. Find the value of cos 810°



$810° = 360 + 360 + 90$

$\cos\theta = \frac{x}{r}$

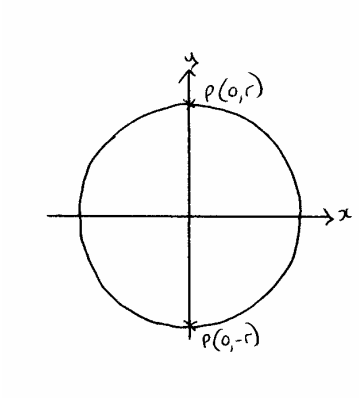
$\cos 810° = \frac{0}{r}$

$\cos 810° = 0$

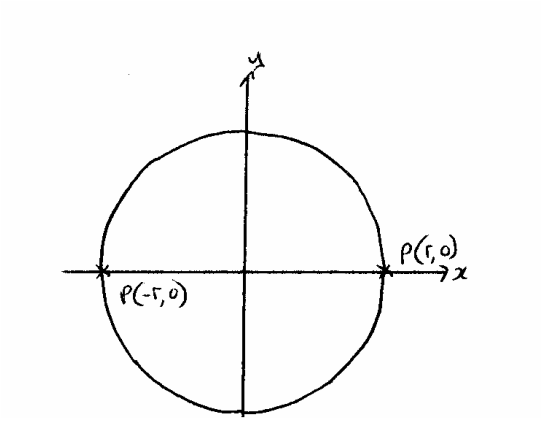
For Tan

a) Tan is indeterminate when θ is an odd multiple of 90°.

When $y = 0$ $\tan\theta = 0$. This is because when P is at (r ,0) or (− r, 0)

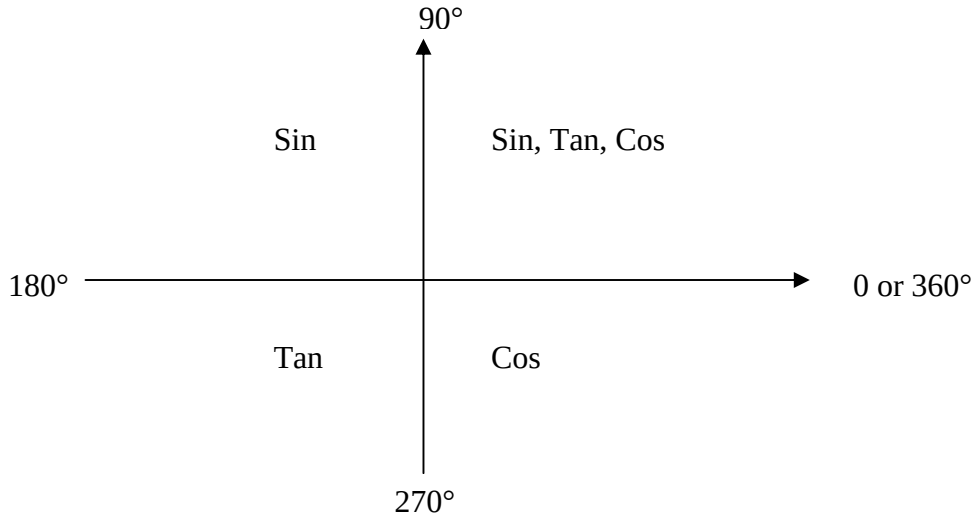


b) Tan θ = 0 When θ is 0° or an even multiple of 90°



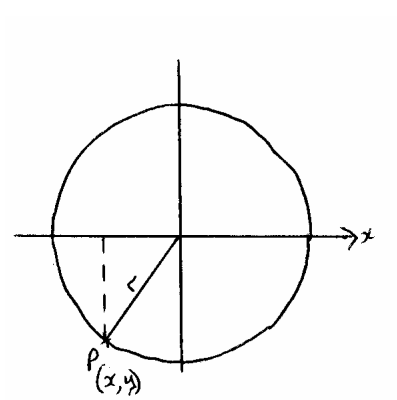
Expressing Angles in Term of Equivalent Acute Trigonometric Ratios

Angle	Positive	Negative
0-90	All	None
90-180	Sin	Cos,Tan
180-270	Tan	Sin,Cos
270-360	Cos	Sin, Tan



Learn the saying "All Sinners Tan Cos they can!!

Why?



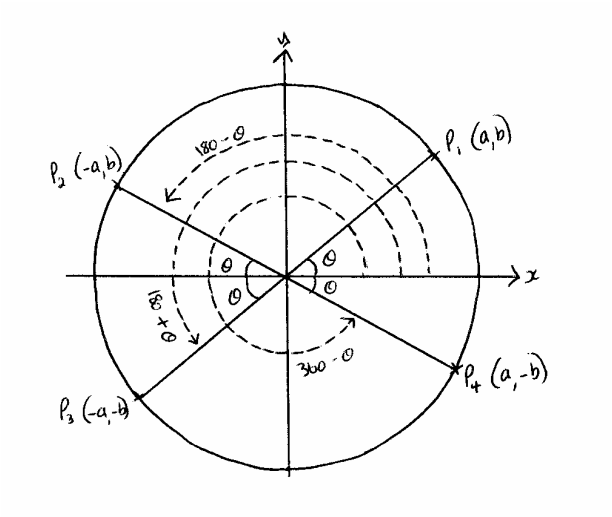
In 3rd Quadrant: –

$$\sin\theta = \frac{y}{r} \qquad = \frac{-ve}{+ve} = -ve$$

$$\cos\theta = \frac{x}{r} \qquad = \frac{-ve}{+ve} = -ve$$

$$\tan\theta = \frac{y}{x} \qquad = \frac{-ve}{-ve} = +ve$$

Similar Angles



for $\sin\theta = \frac{y}{r}$ so: –

$\sin\theta = \frac{b}{r}$ $\qquad\qquad$ $\sin\theta = \sin 180 - \theta$

$\sin(180 - \theta) = \frac{b}{r}$ $\qquad\qquad$ $\sin 180 + \theta = \sin 360 - \theta$

$\sin(180 + \theta) = -\frac{b}{r}$ $\qquad\qquad$ $\sin 180 + \theta = -\sin\theta$

$\sin(360 - \theta) = -\frac{b}{r}$ $\qquad\qquad$ $\sin 360 - \theta = -\sin\theta$ etc

The results for sine, cosine and tangent are:-

Quadrant 2

$\sin(180 - \theta) = \sin\theta$

$\cos(180 - \theta) = -\cos\theta$

$\tan(180 - \theta) = -\tan\theta$

Notice the connection with the quadrants.

It is telling us where the answers will be positive or negative.

Quadrant 3

$\sin(180 + \theta) = -\sin\theta$

$\cos(180 + \theta) = -\cos\theta$

$\tan(180 + \theta) = \tan\theta$

Quadrant 4

$\sin((360 - \theta) = -\sin\theta$

$\cos(360 - \theta) = \cos\theta$

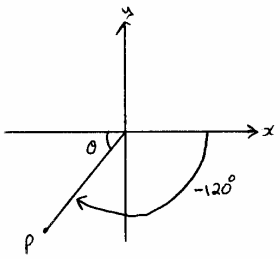
$\tan(360 - \theta) = -\tan\theta$

Example 1. Express in terms of trigonometric ratios of acute angles, the angle sin (-120°).

In the 3rd quadrant sin is negative

$= -\sin\theta$

$= -\sin 60°$

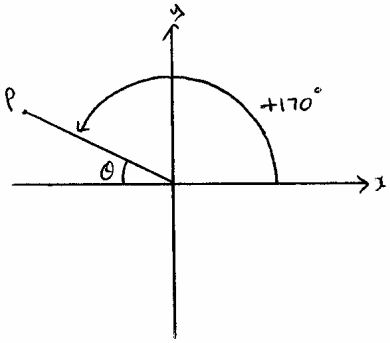


Example 2. Express in terms of trigonometric ratios of acute angles, the angle tan (530°)

In the 2nd quadrant tan is negative

$= -\tan\theta$

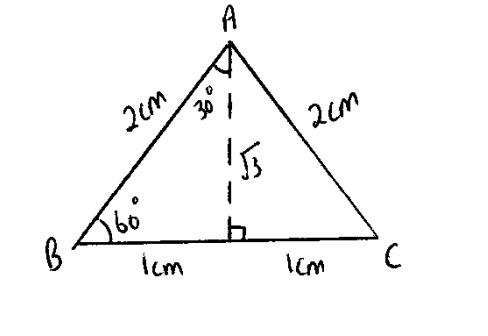
$= -\tan 10°$



Finding Exact Values of Trigonometric Ratios

How to find 30° and 60° angles

Take an equilateral triangle with sides 2cm
(you could do this with any equilateral triangle)



$AD = \sqrt{2^2 - 1^2}$

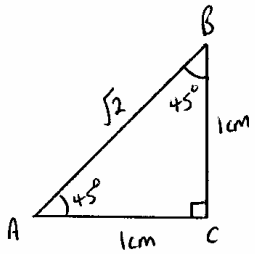
$AD = \sqrt{3}\ cm$

$\therefore \quad \sin 30° = \frac{1}{2} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2} \qquad \tan 60° = \sqrt{3}$

How to find a 45° angle

Take an isosceles right angled triangle with sides 1cm
(again you can use any isosceles triangle)

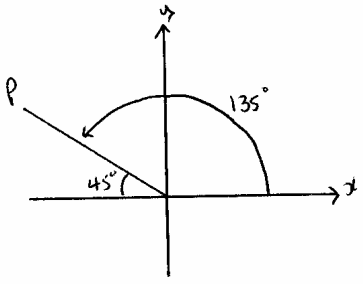


$AB = \sqrt{1^2 - 1^2}$

$AB = \sqrt{2}\ cm$

$\therefore \quad \sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan 45° = \frac{1}{1} = 1$

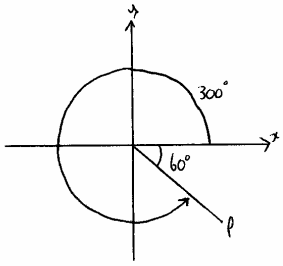
Example 1. Find the exact value of sin 135°



$\sin 135° = \sin 45°$ sin is positive in 2nd quadrant

$= \frac{\sqrt{2}}{2}$

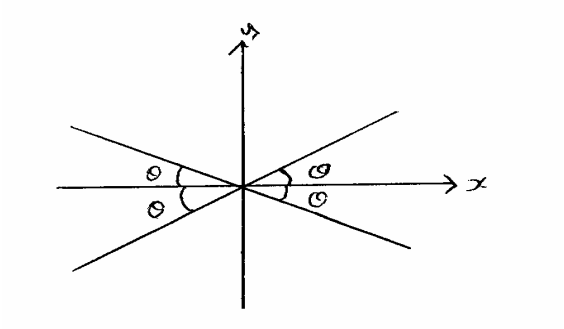
Example 2. Find the exact value of tan 300°



$\tan 300° = -\tan\theta$ tan is negative in 4th quadrant

$= -\tan 60°$

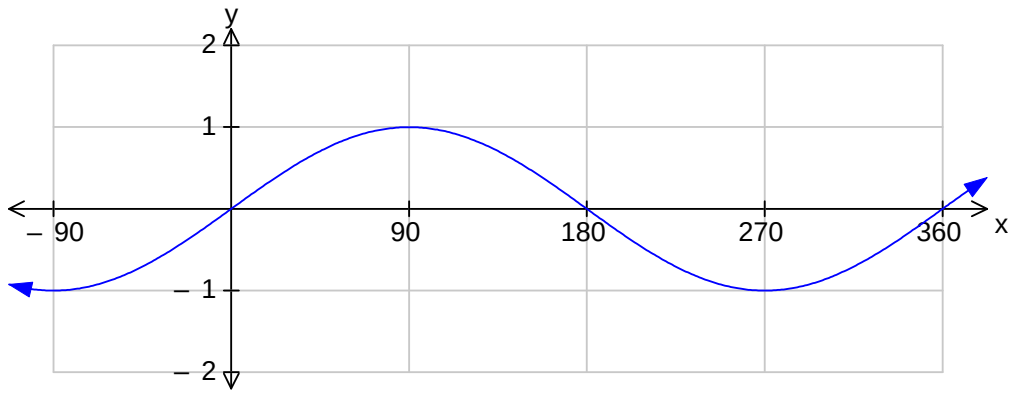
Remember to first work out the value of θ, then work out whether the quadrant is positive or negative.



Recognising Trigonometric Graphs

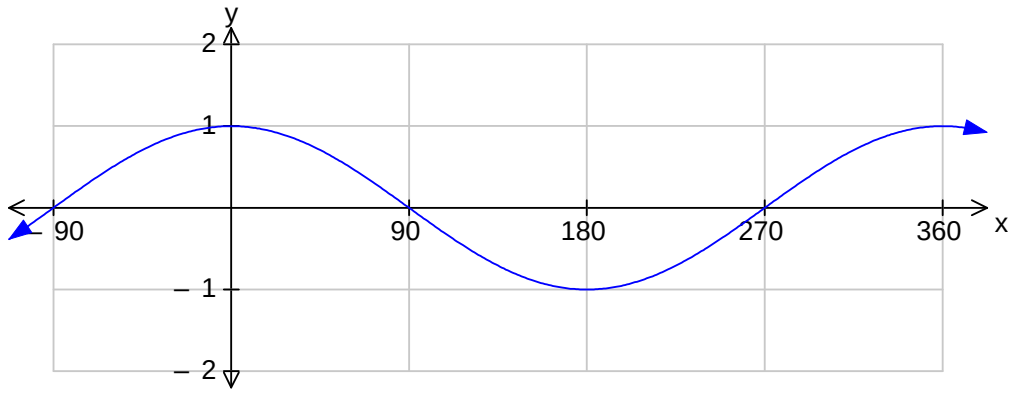
y = sin *x*

The graph is periodic, the period is 360° or 2π radians, the graph is symmetrical about θ = 90°



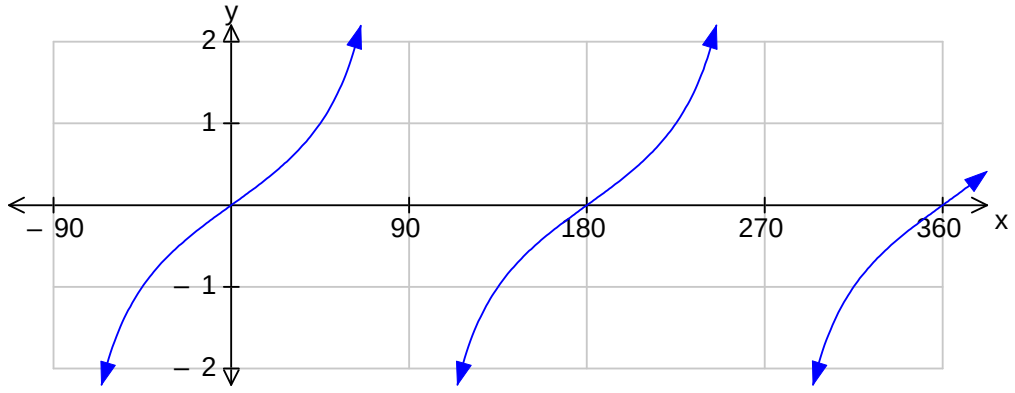
y= cos *x*

The graph is periodic, the period is 360° or 2π radians, the graph is the same as the cos graph which has been translated 90° to the left, the graph is symmetrical about θ = 0°



y = tan *x*

The graph is periodic, the period is 180° or 2π radians, asymptotes occur at 90° and at odd multiples of 90° there after.



Transformation of Trigonometric Graphs

These follow the same rules as we had for normal graphs.

a) $y = \sin x + 1$

Move the graph up 1 = add 1 to the y coordinate

b) $y = \cos(x + 90°)$

Move the graph left 90° = subtract 90° to the x coordinate

c) $y = 3\cos x$

Stretch the graph vertically by 3 = multiply the y coordinate by 3

d) $y = \sin 2x$

Stretch the graph horizontally by ½ = multiply the x coordinate by 2

e) $y = -\tan x$

Reflect the graph in the x-axis = change the sign of the y coordinate

f) $y = \cos(-x)$

Reflect the graph in the y-axis = change the sign of the x coordinate