Finding the Partial Sum of a Geometric Series

General formula for a partial sum is:-

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{where r > 1}$$

or $$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{where r < 1}$$

Why?

If $$S_n = a + ar + ar^2 + ar^3 + ar^4 + ...ar^{n-1}$$

$(\times r)$ $$r S_n = ar + ar^2 + ar^3 + ar^4 + ar^5 + ...ar^n$$

(subtract equations) $rS_n - S_n = -a + ar^n$ or $S_n - rS_n = a - ar^n$

$$S_n(r - 1) = a(r^n - 1) \qquad S_n(1 - r) = a(1 - r^n)$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_n = \frac{a(1 - r^n)}{1 - r}$$

Example 1. Find the sum of the first 9 terms of the geometric series 18+12+8+……

$a = 18 \quad r = \frac{12}{18} \quad n = 9$ using $S_n = \frac{a(r^n - 1)}{r - 1}$

$$S_9 = \frac{18(0{\cdot}6666^9 - 1)}{0{\cdot}6666 - 1}$$

$$S_9 = \frac{18(0{\cdot}93988)}{0{\cdot}333333}$$

$$S_9 = 53{\cdot}9945$$

$$S_n = 54 \ (2sf)$$

Example 2. Find the sum of the following geometric series 0.5 + 1 + 2 + 4 ……..+1024

$a = 0{\cdot}5 \ r = 2 \ u_n = 1024 \ n = n$ using $u_n = ar^{n-1}$

$$1024 = 0{\cdot}5 \times 2^{n-1}$$

First we need to know how many terms we are adding together.

$$\log 2048 = \log 2^{n-1}$$

$$\log 2048 = (n - 1)\log 2$$

$$\frac{\log 2048}{\log 2} = n - 1$$

$$11 = n - 1$$

$$12 = n$$

This answer means we want to add together the first 12 terms.

$a = 0{\cdot}5 \quad r = 2 \quad n = 12 \qquad \text{using} \quad S_n = \dfrac{a(r^n - 1)}{r - 1}$

$$S_{12} = \frac{0{\cdot}5(2^{12} - 1)}{2 - 1}$$

$$S_{12} = \frac{0{\cdot}5(4095)}{1}$$

$$S_{12} = 2047{\cdot}5$$

$\therefore$ the sum of the first 12 terms is 2047.5

Example 3. Find $\sum_{r=1}^{10} 3 \times 2^r$

Using this information the sequence begins 6, 12, 24 ($3x2^1$, $3x2^2$ etc)

$a = 6 \quad n = 10 \quad r = 2$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{6(2^{10} - 1)}{2 - 1}$$

$$S_{10} = \frac{6 \times 1023}{1}$$

$$S_{10} = 6138$$