

Defining a Geometric Sequence

The general term for a geometric sequence is:-

$$u_n = ar^{n-1} \quad \text{where } a = \text{1st term,}$$

r = common difference

n = term number

U_n = term you are calculating

\therefore sequence goes $a, ar, ar^2, ar^3, ar^4, \text{ etc.}$

Example 1. The first 4 terms of a geometric sequence are 4, 12, 36, 108. Find the 13th term.

$$a = 4, r = 3, n = 13 \quad \text{Using } u_n = ar^{n-1}$$

$$\begin{aligned} U_{13} &= 4 \times 3^{13-1} \\ &= 4 \times 3^{12} \\ &= 2125764 \end{aligned}$$

Example 2. a) Find the common ratio of the geometric sequence 3, 4.5, 6.75, 10.125

$$\text{Common ratio} = \frac{U_{n+1}}{u_n}$$

$$r = \frac{4.5}{3}$$

$$r = 1.5$$

b) Find the 20th term

$$a = 3, r = 1.5, n = 20 \quad \text{Using } u_n = ar^{n-1}$$

$$\begin{aligned} U_{20} &= 3 \times 1.5^{20-1} \\ &= 3 \times 1.5^{19} \\ &= 6650.51346 \end{aligned}$$

c) Find the n th term

$$a = 3, r = 1.5, n = n \quad \text{Using } u_n = ar^{n-1}$$

$$\begin{aligned} U_n &= 3 \times 1.5^{n-1} \\ &= 3 \times \left(\frac{3}{2}\right)^{n-1} \end{aligned}$$

Example 3.

If the 6th term of a geometric sequence is 32 and the 3rd term is 4. Find the first term and the common ratio.

Using $u_n = ar^{n-1}$

$$u_6 = ar^{6-1} \qquad u_3 = ar^{3-1}$$

$$u_6 = 32 \qquad u_3 = 4$$

$$\therefore ar^5 = 32 \qquad ar^2 = 4$$

By comparing the ratios

$$\frac{ar^5}{ar^2} = \frac{32}{4}$$

$$r^3 = 8$$

$$\therefore r = 2$$

Substitute the value of r into one of your equations

$$r = 2 \qquad ar^2 = 4$$

$$a \times 2^2 = 4$$

$$4a = 4$$

$$a = 1$$

So the common ratio is 2 and the first term is 1