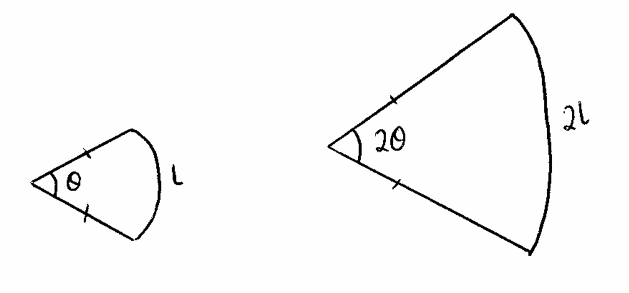
<u>Finding Arc Lengths</u>

The length of an arc is always proportional to the angle at the centre of the arc and the radius of the arc. So, if 2 arcs have the same radius but one has an angle twice the size of the other, it means one arc length will be twice the size of the other.



Formula for finding an arc length: – $l = r\theta$ where l = arc length,

r = radius

θ = angle at the centre

Why?

$$\frac{l}{2\pi r} = \frac{\theta}{2\pi}$$

$$(\times 2\pi r) \qquad l = \frac{2\theta\pi r}{2\pi}$$

$$l = r\theta$$

$\frac{\text{Length of arc}}{\text{Circumference}} = \frac{\text{angle at the centre}}{\text{total angle at centre}}$

↑ ↑

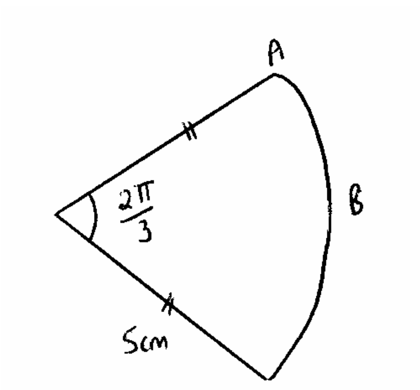
Ratio of sides = Ratio of angles

Example 1. Find the length of the arc ABC.

$l = r\theta \qquad r = 5cm \quad \theta = \frac{2\pi}{3}$

$l = 5 \times \frac{2\pi}{3}$

$l = \frac{10\pi}{3}$ or $10{\cdot}47cm$



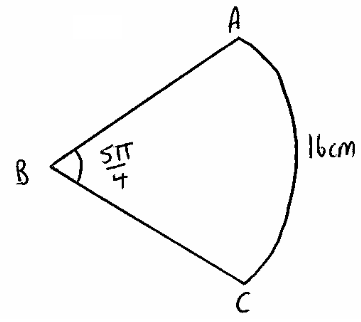
Example 2. Find the radius of the sector ABC

$l = r\theta \qquad r = 16cm \quad \theta = \frac{5\pi}{4}$

$16 = r \times \frac{5\pi}{4}$

$16 \times \frac{4}{5\pi} = r$

$\frac{64\pi}{5\pi} = r$ or $r = 4{\cdot}07cm$



Example 3. An arc AB or a circle, with centre o and radius r cm, subtends an angle of θ radians at O. The perimeter of the sector AOB is P cm.
Express r in terms of θ.

$p = (2 \times \textit{radius}) + \text{arc length}$

$p = 2r + r\theta$

$\frac{p}{2} + \theta = r$