

Expanding Polynomials Using the Binomial Expansion

You need to remember these facts...

$$(1 + x)^n = \binom{n}{0} 1^n + \binom{n}{1} 1^{n-1} x^1 + \binom{n}{2} 1^{n-2} x^2 \dots \dots \dots \text{etc}$$

$$\text{or} \quad = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots \dots \dots \text{etc}$$

Example 1.


$$\begin{aligned}(1 + 2x)^5 &= 1 + (5 \times 2x) + \frac{5 \times 4 \times (2x)^2}{2!} + \frac{5 \times 4 \times 3 \times (2x)^3}{3!} + \\ &\quad \frac{5 \times 4 \times 3 \times 2 \times (2x)^4}{4!} + \frac{5 \times 4 \times 3 \times 2 \times 1 \times (2x)^5}{5!} + \\ &= 1 + 10x + (10 \times 4x^2) + (10 \times 8x^3) + (5 \times 16x^4) + (1 \times 32x^5) \\ &= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5\end{aligned}$$

Taking Out the Common Factors

The Binomial Expansion only works for $(1 + x)^n$ so sometimes you need to take out a factor first.

$$(2 + x)^3 = \left(2 \left(1 + \frac{x}{2} \right) \right)^3$$

$$= 2^3 \left(1 + \frac{x}{2} \right)^3 \longleftarrow \text{Expand as normal}$$


Remember to
multiply by this at
the end