

Factorial Notation

A factorial notation looks like this $n! = n \times (n - 1) \times (n - 2) \dots$

Example 1. $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

To find the number of ways of choosing r items from a group of n items is written as

$${}^n C_r \text{ or } \binom{n}{r}$$

This is calculated by $\frac{n!}{r!(n-r)!}$

Example 2. Find ${}^5 C_3$

$$\begin{aligned} &= \frac{5!}{2! \times 3!} \\ &= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} \\ &= \frac{120}{2 \times 6} \\ &= 10 \end{aligned}$$

Example 3. Find $\binom{7}{4}$

$$\begin{aligned} &= \frac{7!}{3! \times 4!} \\ &= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} \\ &= 35 \end{aligned}$$

Example 4. 4 People need to sit down but 2 want to sit together. How many different combinations are there?

Using ${}^n C_r = {}^4 C_2$

$$\begin{aligned} &= \frac{4!}{2!2!} \\ &= \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} \\ &= 6 \end{aligned}$$

The Connection between Combination Notation and Pascal's Triangle

Look at this:-

$${}^4C_0 = 1 \quad {}^4C_1 = 4 \quad {}^4C_2 = 6 \quad {}^4C_3 = 4 \quad {}^4C_4 = 1$$

These values are the same as the Index 4 line

$$\therefore \text{6th line would be } \begin{pmatrix} 6 \\ 0 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$