Equation of a Circle

An equation of a circle is always in the form $(x-a)^2+(y-b)^2=r^2$
where r is the radius and (a,b) is the centre of the circle.

Example 1. If a circle has a radius of 7 and a centre at (2,6), what is the equation of the circle?

$(x-a)^2+(y-b)^2=r^2$ where $a=2, b=6, r=7$

$\therefore \quad (x-2)^2+(y-6)^2=7^2$

$(x-2)^2+(y-6)^2=49$

The equation of the cirlce is $(x-2)^2+(y-6)^2=49$

Example 2. Given the equation $(x-2\sqrt{3})^2+(y+\sqrt{7})^2=144$, find the radius of the centre of the circle.

$(x-a)^2+(y-b)^2=r^2$ where $a=2\sqrt{3}$, $b=-\sqrt{7}$, $r^2=144$

$\therefore$ centre is $(2\sqrt{3}, -\sqrt{7})$

$r^2=144$

$r=\pm 12$ as radius cannot be negative we can ingnore the negative value

$\therefore r=12$

Example 3. Prove that (1,2) lies on the circumference of the circle which has the equation

$(x-2)^2+(y+3)^2=26$ when $x=1$ $y=2$

$(1-2)^2+(2+3)^2=26$

$(-1)^2+(5)^2=26$

$1+25=26$

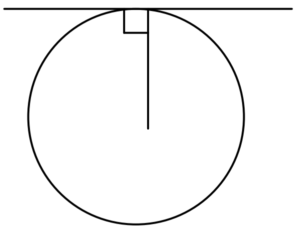
$26=26$

$\therefore$ (1,2) lies on the circumference of the circle

Tangents

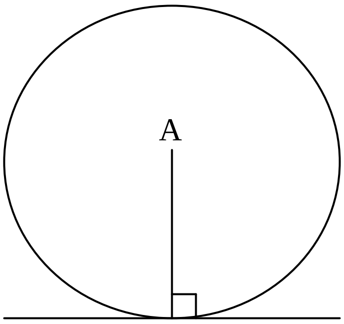
The angle between the tangent and a radius is 90°. A tangent only touches at one point.

This circle theorem is often used in questions as it can relate closely to perpendicular bisectors.



Example 1. The line $4x - 3y - 40 = 0$ touches the circle $(x-2)^2 + (y-6)^2 = 100$ at P(10,0). Show that the radius at P is perpendicular to the line.

This mean the centre A is (2,6)



$$\text{Gradient of AP} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 1}{2 - 10}$$

$$= \frac{6}{-8}$$

$$\therefore \text{ gradient of AP is } -\frac{3}{4}$$

Gradient of tangent $4x - 3y - 40 = 0$

$$4x - 40 = 3y$$

$$\frac{4x - 40}{3} = y$$

$$\therefore \text{gradient of tangent is } \frac{4}{3}$$

Using $m_1 \times m_2 = -1$ where $m_1 = -\frac{3}{4}$ and $m_2 = \frac{4}{3}$

$$-\frac{3}{4} \times \frac{4}{3} = -1$$

$\therefore$ lines are perpendicualr