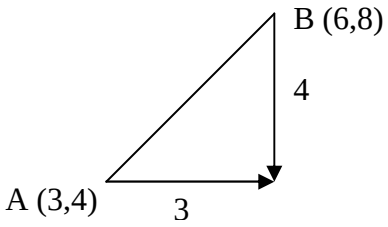
Distance Between Two Points

To work out the distance between two points we use Pythagoras

Midpoint of AB = (2,2)



The formula for the distance between two points is:-

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where (x_1, y_1) and (x_2, y_2) are 2 given points on the line

Example 1. PQ is the diameter of a circle where p(− 1,3) and Q(6, − 3) .

Find the radius of the circle

First we need to remember that Radius = half the Diameter

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(6 - -1)^2 + (-3 - 3)^2}$$

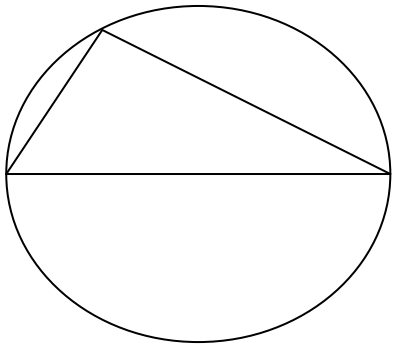
$$PQ = \sqrt{(7^2 + (-6)^2)}$$

$$PQ = \sqrt{85}$$

$$\textbf{Radius} = \frac{\sqrt{85}}{2}$$

Angles in a semicircle

An angle in a semicircle is always 90° when one side of the triangle is the diameter and all 3 sides sit on the circumference of the circle



Example 2. *The points A(2,6) , B(5,7) and C(8, − 2) lie on a circle. Show that ABC is a right angled triangle and find the area of the triangle*

$$\text{Length AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{AB} = \sqrt{(5 - 2)^2 + (7 - 6)^2}$$

$$\text{AB} = \sqrt{3^2 + 1^2}$$

$$\text{AB} = \sqrt{10}$$

$$\text{Length BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{BC} = \sqrt{(8 - 5)^2 + (-2 - 7)^2}$$

$$\text{BC} = \sqrt{3^2 + (-9)^2}$$

$$\text{BC} = \sqrt{90}$$

$$\text{Length AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{AC} = \sqrt{(8 - 2)^2 + (-2 - 6)^2}$$

$$\text{AC} = \sqrt{6^2 + (-8)^2}$$

$$\text{AC} = \sqrt{100}$$

$$\text{AC} = 10$$

$\therefore$ Using pythagoras to prove ABC is a right angled triangle

$$\text{AC}^2 = \text{AB}^2 + \text{BC}^2$$

$$10^2 = (\sqrt{10})^2 + (\sqrt{90})^2$$

$$100 = 10 + 90$$

$$100 = 100$$

This proves the triangle is a right angled triangle

$$\text{Area of a triangle} = \frac{1}{2} \times \textit{base} \times \textit{height}$$

$$A = \frac{1}{2} \times \text{AB} \times \text{BC}$$

$$A = \frac{1}{2} \times \sqrt{10} \times \sqrt{90}$$

$$A = \frac{1}{2} \times \sqrt{10} \times \sqrt{9} \times \sqrt{10}$$

$$A = 15 \text{ units}^2$$