Changing the Base of a Logarithm

The Change of base rule is $\log_a x = \dfrac{\log_b x}{\log_b a}$

if we used $\log_a b$ then it would equal $\dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$

So another rule is $\log_a b = \dfrac{1}{\log_b a}$

Example 1. *Solve* $\log_5 9$

$$\log_5 9 = \frac{\log_{10} 9}{\log_{10} 5}$$

$$= 1{\cdot}365 \text{ (4sf)}$$

Another method is to use what we know already.....

$$\log_5 9 = x$$

$$5^x = 9$$

$$\log_{10} 5^x = \log_{10} 9$$

$$x \log_{10} 5 = \log_{10} 9$$

$$x = \frac{\log_{10} 9}{\log_{10} 5}$$

$$x = 1{\cdot}365 \text{ (4sf)}$$

Example 2. Solve $\log_2 x = 8 + 9\log_x 2$

$$\log_2 x = 8 + 9\log_x 2$$

$$0 = 9\log_2 x - \log_2 x + 8 = 0$$

as $9\log_x 2 = \dfrac{9}{\log_2 x}$

$$\therefore\ 0 = \frac{9}{\log_2 x} - \log^2 x + 8$$

let $\log_2 x = y$

so $\dfrac{9}{y} - y + 8 = 0$

$(\times y)\ 9 - y^2 + 8y = 0$

$$0 = y^2 - 8y - 9$$

$$0 = (y - 9)(y + 1)$$

$$y = 9 \text{ or } y = -1$$

this means $\log_{2x} = 9$ or $\log_2 x = -1$

$$2^9 = x \qquad 2^{-1} = x$$

$$x = 512 \qquad x = \frac{1}{2}$$