Laws of Logarithms

Here are the laws of logarithms; they follow closely the power laws.

1. $\log_a xy = \log_a + \log_a y$ multiplication law

2. $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$ division law

3. $\log_a (x)^k = k \log_a x$ power law

4. $\log_a \left(\frac{1}{x} \right) = - \log_a x$

because $x^{-1} = \frac{1}{x}$ $\therefore \log_a \left(\frac{1}{x} \right) = \log_a x^{-1}$

$= - 1 \log_a x$

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Example 1. Simplify $\log_3 8 + \log_3 25 - \log_3 4$

$= \log_3 \left(\frac{8 \times 25}{4} \right)$

$= \log_3 50$

Example 2. Simplify $\log_5 6 + \log_5 12 - 3 \log_5 2$

$= \log_5 6 + \log_5 12 - \log_5 2^3$

$= \log_5 6 + \log_5 12 - \log_5 8$

$= \log_5 \left(\frac{6 \times 12}{8} \right)$

$= \log_5 9$

Example 3. Expand the following and write in terms of $\log_a x$, $\log_a y$, and $\log_a z$

a) $\log_a \left(\frac{x}{yz} \right) = \log_a x - \log_a y - \log_a z$

b) $\log_a \left(\frac{x^2}{y^3} \right) = \log_a x^2 - \log_a y^3$

$= 2\log_a x - 3 \log_a y$

c) $\log_a \sqrt{axy} = \log_a a^{\frac{1}{2}} + \log_a x^{\frac{1}{2}} + \log_a y^{\frac{1}{2}}$

$= \frac{1}{2} \log_a a + \frac{1}{2} \log_a x + \frac{1}{2} \log_a y$

$= \frac{1}{2} + \frac{1}{2} \log_a x + \frac{1}{2} \log_a y$

Remember $\log_a a = 1$

as $\log_a a = x$

$\therefore \quad a^x = a$

as $a^1 = a$

then $x = 1$