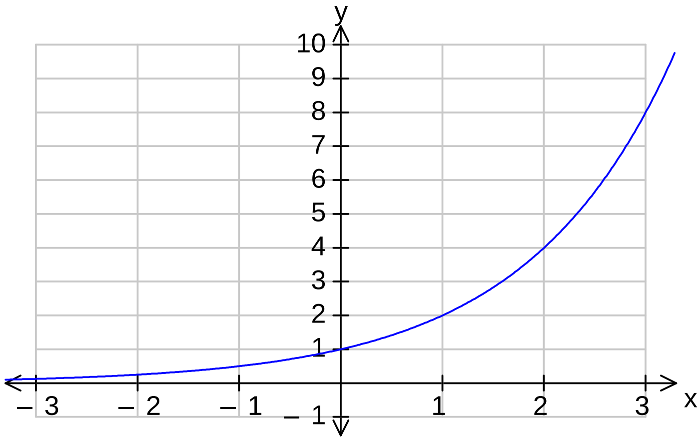
Exponential Function

An exponential graph is in the form $y = a^x$

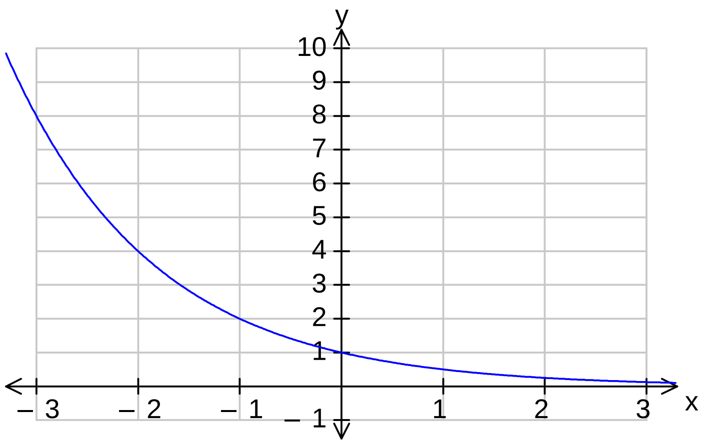
Example 1. *Draw the graph of* $y = 2^x$



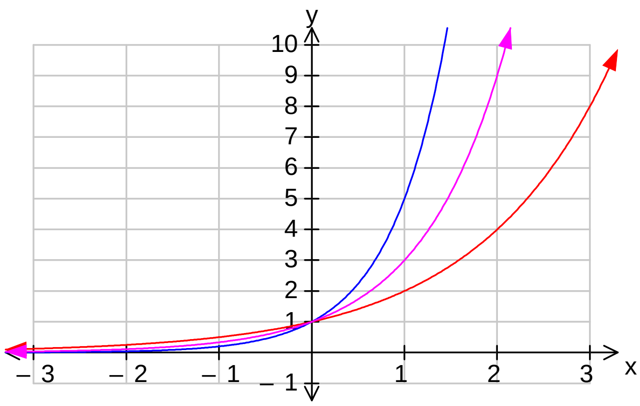
This is what the exponential graph always looks like when a > 0. The x-axis is an asymptote and the graph always passes through the point (0,1)

An exponential graph in the form $y=a^x$ where $(0 < a < 1)$ is the reflection of the normal exponential graph in the y axis. It still passes through the point (0,1) but it goes the other way.

Example 2. *Draw the graph of* $y = 0{\cdot}5^x$

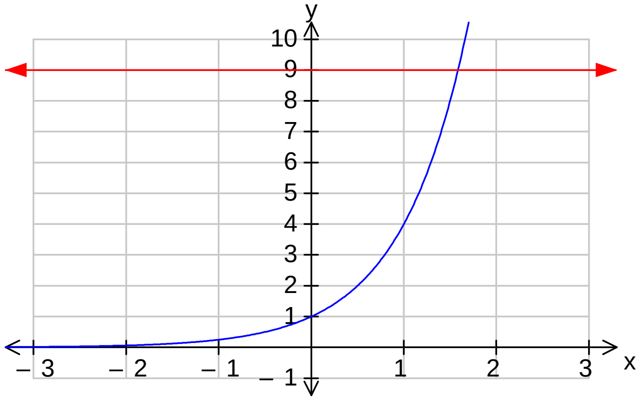


As the value of a gets bigger the graph gets steeper



Solving an equation using your graph

Example 1. *Solve* $4^x = 9$ *using a graph*



x = approximately 1.5 according to the graph

Writing an Expression as a Logarithm

$\log_a n = x$ *means* $a^x = n$ where a is called the base of the logarithm

We use this fact to write an equation as a logarithm so that we can solve it.

Example 1.

$\log_{10} 100 = 2$

$10^2 = 100$

Example 2.

$2^5 = 32$

$Log_2 32 = 5$

Example 3.

Find the value of $Log_3 81$

$\log_3 81 = x$

$3^x = 81$

$x = 4$ as $3^4 = 81$

Example 4.

Find the value of x if $Log_x 64 = 3$

$Log_x 64 = 3$

$x^3 = 64$

$x = 4$ as $4^3 = 64$

Example 5.

Solve $Log_2 x = 3{\cdot}5$

$Log_2 x = 3{\cdot}5$

$2^{3\cdot5} = x$

$x = 11{\cdot}3 \ (1dp)$

<u>**Using your calculator**</u>

Calculators use base 10, so if your expression is in base 10 we can simply type it in.

Example 1. $\text{Log}_{10} 100$

Type in [Log] 100 [=]

Answer should be 2

Example 2. $\text{Log } 10^{x} = 500$

$\text{Log}_{10} 500 = x$

Type in [Log] 500 [=]

Answer should be 2.70 (3sf)

Laws of Logarithms

Here are the laws of logarithms; they follow closely the power laws.

1. $\log_a xy = \log_a + \log_a y$ multiplication law

2. $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$ division law

3. $\log_a (x)^k = k \log_a x$ power law

4. $\log_a \left(\frac{1}{x} \right) = -\log_a x$

because $x^{-1} = \frac{1}{x}$ $\therefore \log_a \left(\frac{1}{x} \right) = \log_a x^{-1}$

$$= -1 \log_a x$$

$$= -\log_a x$$

Example 1. Simplify $\log_3 8 + \log_3 25 - \log_3 4$

$$= \log_3 \left(\frac{8 \times 25}{4} \right)$$

$$= \log_3 50$$

Example 2. Simplify $\log_5 6 + \log_5 12 - 3 \log_5 2$

$$= \log_5 6 + \log_5 12 - \log_5 2^3$$

$$= \log_5 6 + \log_5 12 - \log_5 8$$

$$= \log_5 \left(\frac{6 \times 12}{8} \right)$$

$$= \log_5 9$$

Example 3. Expand the following and write in terms of $\log_a x$, $\log_a y$, and $\log_a z$

a) $\log_a \left(\frac{x}{yz} \right) = \log_a x - \log_a y - \log_a z$

b) $\log_a \left(\frac{x^2}{y^3} \right) = \log_a x^2 - \log_a y^3$

$= 2\log_a x - 3 \log_a y$

c) $\log_a \sqrt{axy} = \log_a a^{\frac{1}{2}} + \log_a x^{\frac{1}{2}} + \log_a y^{\frac{1}{2}}$

$= \frac{1}{2} \log_a a + \frac{1}{2} \log_a x + \frac{1}{2} \log_a y$

$= \frac{1}{2} + \frac{1}{2} \log_a x + \frac{1}{2} \log_a y$

Remember $\log_a a = 1$

as $\log_a a = x$

$\therefore$ $a^x = a$

as $a^1 = a$

then $x = 1$

Solving Equations in the Form $a^x = b$

As some calculator can only work in base 10, we need to make all calculations into base 10.

Example 1. Solve the following equations $2^x = 57{\cdot}35$

$$\log_{10} 2^x = \log_{10} 57{\cdot}35$$

$$x \log_{10} 2 = \log_{10} 57{\cdot}35$$

$$x = \frac{\log_{10} 57{\cdot}35}{\log_{10} 2}$$

$$x = 5{\cdot}842 \ (4sf) \ \text{ using a calculator}$$

Example 2. Solve the following equation $5^{x+1} = 3^{x+2}$

$$\log_{10} 5^{x+1} = \log_{10} 3^{x+2}$$

$$(x + 1)\log_{10} 5 = (x + 2)\log_{10} 3$$

$$x \log_{10} 5 + \log_{10} 5 = x \log_{10} 3 + 2 \log_{10} 3$$

$$x(\log_{10} 5 - \log_{10} 3) = \log_{10} 9 - \log_{10} 5$$

$$x = \frac{\log_{10} 9 - \log_{10} 5}{\log_{10} 5 - \log_{10} 3}$$

$$x = 1{\cdot}151 \ (4sf)$$

Example 3. Solve the equation $2^{2x} - 6(2^x) + 5 = 0$

Let $y = 2^x$ as $2^{2x} - 6(2^x) + 5 = 0$

then $y^2 - 6(y) + 5 = 0$

$$y^2 - 6y + 5 = 0$$

$$(y - 5)(y - 1) = 0$$

$$y = 5 \text{ or } y = 1$$

If $y = 5$ then $2^x = 5$

$$\log_{10} 2^x = \log_{10} 5$$

$$x \log_{10} 2 = \log_{10} 5$$

$$x = \frac{\log_{10} 5}{\log_{10} 2}$$

$$x = 2{\cdot}32 \ (3sf)$$

if y = 1 then $2^x = 1$

$$x = 0$$

$\therefore$ two solutions are 0 and 2.32

Changing the Base of a Logarithm

The Change of base rule is $\log_a x = \dfrac{\log_b x}{\log_b a}$

if we used $\log_a b$ then it would equal $\dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$

So another rule is $\log_a b = \dfrac{1}{\log_b a}$

Example 1. *Solve* $log_5 9$

$$log_5 9 = \frac{log_{10} 9}{log_{10} 5}$$

$$= 1{\cdot}365 \ (4sf)$$

Another method is to use what we know already.....

$$log_5 9 = x$$

$$5^x = 9$$

$$log_{10} 5^x = log_{10} 9$$

$$x \, log_{10} 5 = log_{10} 9$$

$$x = \frac{log_{10} 9}{log_{10} 5}$$

$$x = 1{\cdot}365 \ (4sf)$$

Example 2. Solve $log_2 x = 8 + 9 \, log_x 2$

$$log_2 x = 8 + 9 \, log_x 2$$

$$0 = 9 \, log_2 x - log_2 x + 8 = 0$$

as $$9 \, log_x 2 = \frac{9}{log_2 \, x}$$

$$\therefore \quad 0 = \frac{9}{log_2 \, x} - log^2 x + 8$$

let $log_2 x = y$

so $$\frac{9}{y} - y + 8 = 0$$

$$(\times y) \quad 9 - y^2 + 8y = 0$$

$$0 = y^2 - 8y - 9$$

$$0 = (y - 9)(y + 1)$$

$$y = 9 \text{ or } y = -1$$

this means $log_{2x} = 9$ or $log_2 x = -1$

$$2^9 = x \qquad 2^{-1} = x$$

$$x = 512 \qquad x = \frac{1}{2}$$