<u>**Pythagoras Theorem**</u>

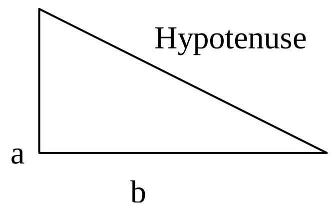
Pythagoras' Theorem is the relationship between the sides of a right angled triangle. It shows that the squares of the two shortest sides are equal to the square of the longest side.

The longest side is called the hypotenuse

The formula is:

$$a^2 + b^2 = c^2$$

where c is always the hypotenuse



Example 1. In a right angled triangle, AB $= 23\cdot5cm$, BC $= 19\cdot3cm$ and $\angle ABC = 90°$. Find the length of AC.

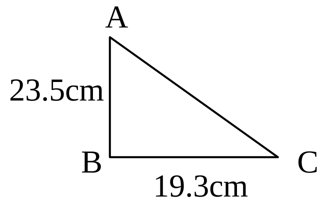
$$a^2 + b^2 = c^2 \qquad \text{where } a = 23\cdot5\ ,\ b = 19\cdot3$$

$$23\cdot5^2 + 19\cdot3^2 = x^2$$

$$552\cdot25 + 372\cdot49 = x^2$$

$$924\cdot75 = x$$

$$x = 30\cdot41cm\ (2dp)$$



You need to remember that if you don't know the longest side you add

$$a^2 + b^2 = c^2$$

If you know the longest side you subtract

$$a^2 = c^2 - b^2$$

<u>Trigonometry</u>

The ratio of any two sides of a right angled triangle will always remain the same if the angles stay the same. From this we can find three ratios:-

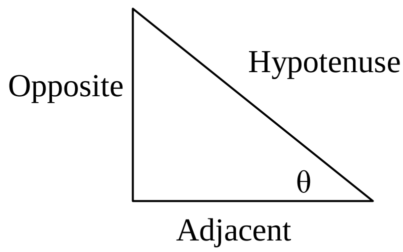
$$\sin\theta = \frac{opp}{hyp} \qquad os\,\theta = \frac{ady}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

It is important the sides are always named relative to the angle given (this does not include the right angle)

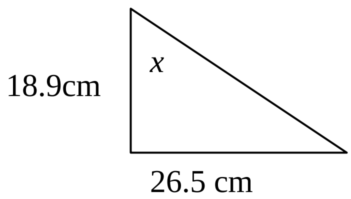
Hypotenuse is always across from the right angle, it never touches it

Opposite faces the given angle but never touches it.

Adjacent is next to the angle and it touches both the right angle and the given angle.



Example 1. Find the size of angle x



$$\tan\theta = \frac{opp}{adj}$$

$$\tan x = \frac{26\cdot5}{18\cdot9}$$

$$\tan x = 1\cdot402$$

$$x = 54\cdot5cm\ (1dp)$$

If x is the angle then work out the fraction and use shift tan

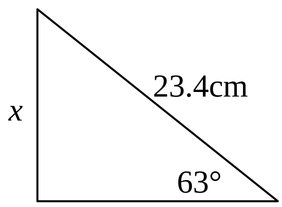
Example 2. Find the size of length x

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 63^\circ = \frac{x}{23\cdot4}$$

$$\sin 63^\circ \times 23\cdot4 = x$$

$$x = 20\cdot85cm$$



If x is at the top of the fractions then you need to multiply

"top means times"

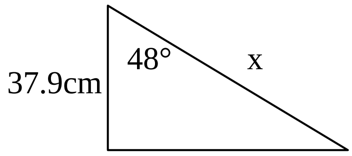
Example 3. Find the size of length x

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 48° = \frac{37\cdot9}{x}$$

$$x = \frac{37\cdot9}{\cos 48}$$

$$x = 56\cdot64cm\ (2dp)$$



If x is at the bottom then you need to divide the number by sin/cos or tan

“bottom means divide”

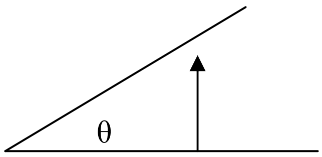
Remember:

If x is at the top then times
If x is at the bottom then divide
If x is the angle then work out the fraction and then shift sin/cos/tan

Angles of Elevation and Depression

Angles of Elevation

This is an angle always taken from the horizontal upwards. It is often used in trigonometry questions to describe the position of the angle.



The angle of elevation is the angle from the horizontal upwards

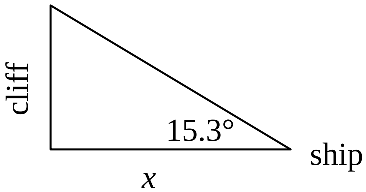
Example 1. A ship (S) is out at sea. The angle of elevation from the ship to the top of a cliff is 15.3°. If the vertical height of the cliff is 680m, how far away from the foot of the cliff is the ship.

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 15{\cdot}3° = \frac{680}{x}$$

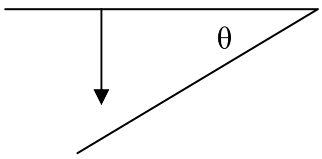
$$x = \frac{680}{\tan 15{\cdot}3}$$

$$x = 2485{\cdot}66m$$



Angles of Depression

This is an angle always taken from the horizontal downwards. It is also often used in trigonometry questions to describe the position of the angle.



The angle of elevation is the angle from the horizontal downwards

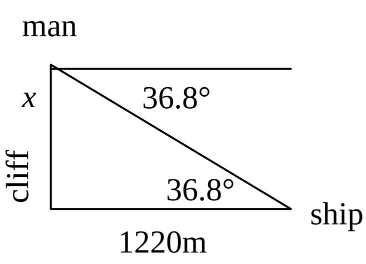
Example 2. A man (M) is standing on the top of a cliff. He is looking out to sea at a boat. The angle of depression from the man to the boat is 36.8°. If the boat is 1220m away from the foot of the cliff, how high is the cliff.

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 36{\cdot}8° = \frac{x}{1220}$$

$$x = \tan 36{\cdot}8 \times 1220$$

$$x = 921{\cdot}68m$$



Sine Rule

The sine rule is:-

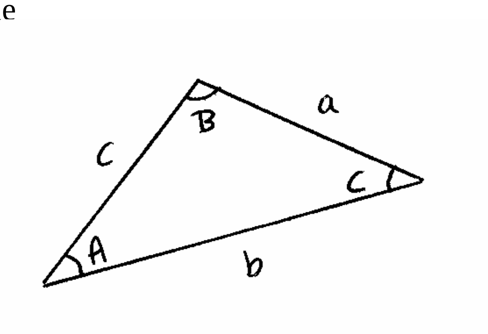
$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$

Choose this formula for an unknown side

or $$\frac{Sin A}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c}$$

Choose this formula for an unknown angle

To use the sine rule you must have a ***complete ratio*** – by that I mean you must know one side and its corresponding angle



How to prove the sine rule using trigonometry.

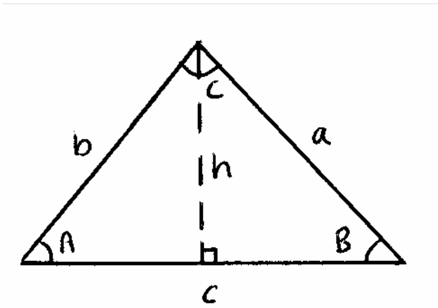
$$Sin\ B = \frac{h}{a} \qquad Sin\ A = \frac{h}{b}$$

$$h = a \sin B \qquad h = b \sin A$$

$$\therefore a\ Sin\ B = b \sin A$$

$$a = \frac{b \sin A}{Sin\ B}$$

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B}$$



Finding an Unknown Length

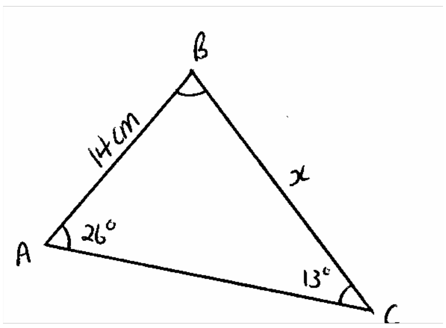
Example 1 *Find the size of BC, given that AB = 14cm , ∠BAC = 26°, ∠ACB = 13°.*

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 26^\circ} = \frac{14}{\sin 13^\circ}$$

$$x = \frac{14}{\sin 13^\circ} \times \sin 26^\circ$$

$$x = 27{\cdot}3cm\ (1dp)$$

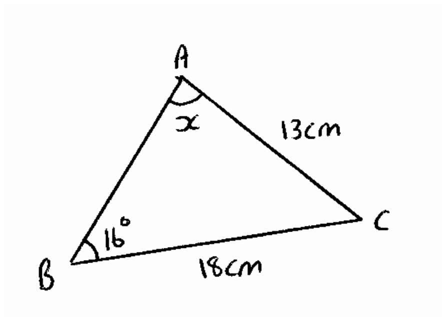


Sine Rule

Finding an unknown Angle

Example 1. Find the size of $\angle BAC$, given that AC = $13cm$, $\angle ABC$ = 16° , and BC = $18cm$

$$\frac{Sin\ A}{a} = \frac{Sin\ B}{b}$$



$$\frac{Sin\ x}{18} = \frac{\sin 16°}{13}$$

$$\sin x = \frac{\sin 16°}{13} \times 18$$

$$\sin x = 0{\cdot}3817$$

$$x = 22{\cdot}4°$$

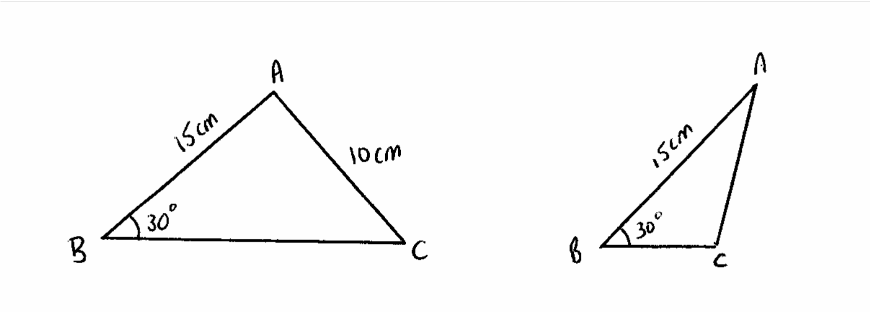
Leave it in the calculator to keep your accuracy

Finding Two Solutions for a Missing Angle

In general if $x = y°$ then x is also equal to $180 - y°$

This is because sometimes you can draw a triangle in 2 different ways.

Example 1. Given triangle ABC, where $\angle ABC = 30°$, AB = 15cm and BC = 10cm



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin x}{15} = \frac{\sin 30°}{10}$$

$$\sin x = \frac{\sin 30°}{10} \times 15$$

$$\sin x = 0{\cdot}75$$

$$x = 48{\cdot}6°$$

Also as $x = 180 - y°$ $\quad x = 180 - 48{\cdot}6$

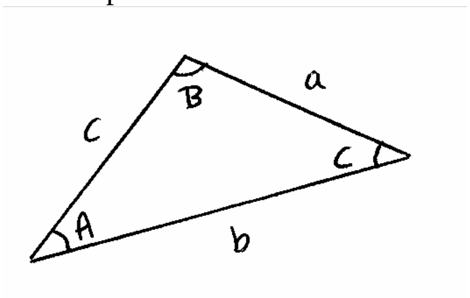
$$x = 131{\cdot}4°$$

so two solutions are $48{\cdot}6°$ and $131{\cdot}4°$

Note: This only occurs if the angle you are finding is larger than the angle given.

The Cosine Rule

This is the last of the triangle formulas. You should only use this if you know all 3 sides or you do not have a complete ratio.



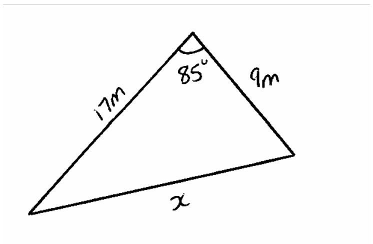
Formulas:

To find a missing angle $\quad Cos\ A = \dfrac{b^2 + c^2 - a^2}{2bc}$

To find a missing side $\quad a^2 = b^2 + c^2 - (2bc\ Cos\ A)$

Finding an Unknown Side

Example 1. Find the value of x



$$a^2 = b^2 + c^2 - (abc\ Cos\ A)$$

$$a^2 = 9^2 + 17^2 - (2 \times 9 \times 17 \times \cos 85)$$

$$a^2 = 343{\cdot}33$$

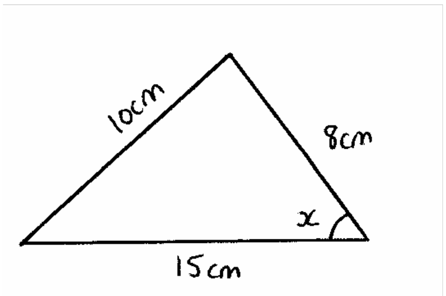
$$a = 18{\cdot}53m\ (2dp)$$

Remember to leave this value in your calculator to keep your accuracy

The Cosine Rule

Finding an Unknown Angle

Example 1. Find the value of x



$$Cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$Cos\ x = \frac{15^2 + 8 - 10}{2 \times 15 \times 8}$$

$$Cos\ x = 0{\cdot}7875$$

$$x = 38{\cdot}05^\circ\ (2dp)$$

Remember to leave this value in your calculator to keep your accuracy

Using all Formulas to Solve Problems

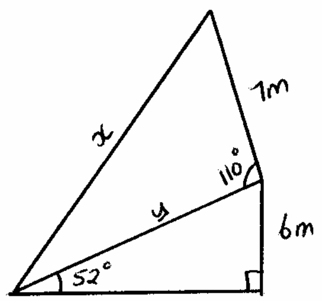
The order in which you should attempt triangle problems is:-

1. Pythagoras
2. Trigonometry — use these two for right angled triangles

3. Sine Rule
4. Cosine Rule — use these two for **non** right angled triangles

Remember also your angle facts such as angles in a triangle equal 180°

Example 1. Find the size of length x

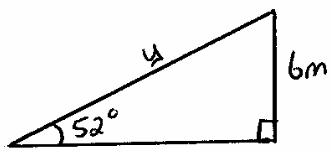


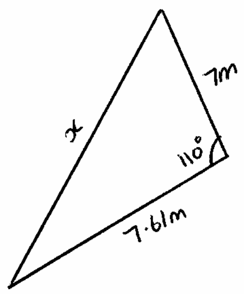
using $\sin\theta = \dfrac{opp}{hyp}$

$\sin = \dfrac{6}{y}$

$y = \dfrac{6}{\sin 52}$

$y = 7{\cdot}61cm$





$a^2 = b^2 + c^2 - (2bc \cos A)$

$x^2 = 7^2 + 7{\cdot}61^2 - (2 \times 7 \times 7{\cdot}61 \times \cos 110)$

$x^2 = 143{\cdot}35$

$x = 11{\cdot}97m \ \ (2dp)$

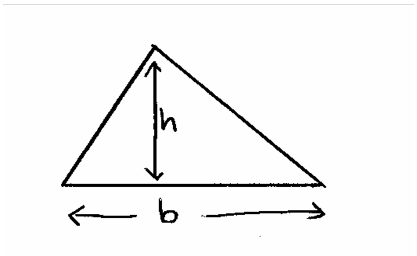
Areas of Triangles

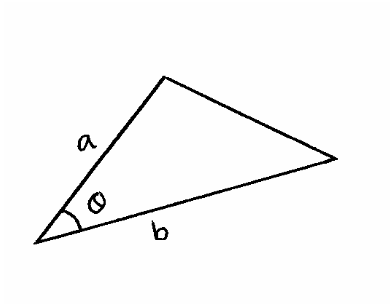
There are 2 formulas for finding the area of a triangle.

Formulas:

Area $= \frac{1}{2}$ x base x perpendicular height or Area $= \frac{1}{2} ab \sin \theta$

$$A = \frac{b \times h}{2}$$



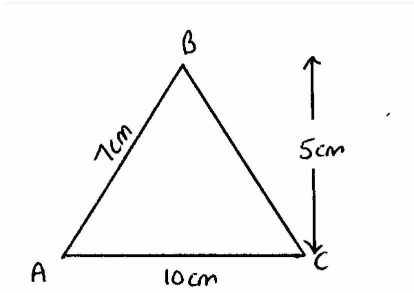


Example 1. Find the area of triangle ABC (diagram not to scale)

$$A = \frac{b \times h}{2}$$

$$A = \frac{10 \times 5}{2}$$

$$A = 25cm^2$$



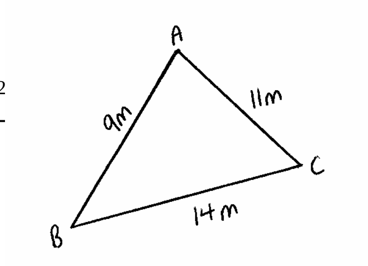
Example 2. Find the area of triangle ABC (diagram not to scale)

$$Cos\,A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$Cos\,\theta = \frac{9^2 + 11^2 - 14^2}{2 \times 9 \times 11}$$

$$Cos\,\theta = 0{\cdot}030303$$

$$\theta = 88{\cdot}26°$$



$$\therefore \quad A = \frac{1}{2} ab\,Sin\,\theta$$

$$A = \frac{1}{2} \times 9 \times 11 \times \sin 88{\cdot}26°$$

$$A = 49{\cdot}48m^2$$