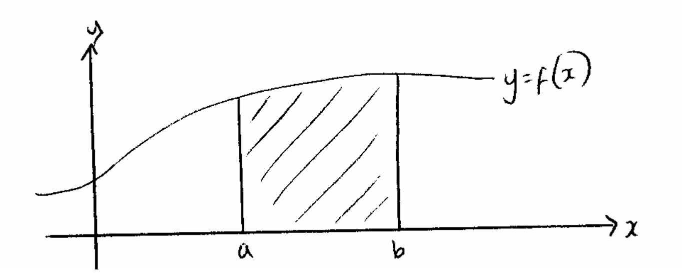
Finding Areas Under Curves

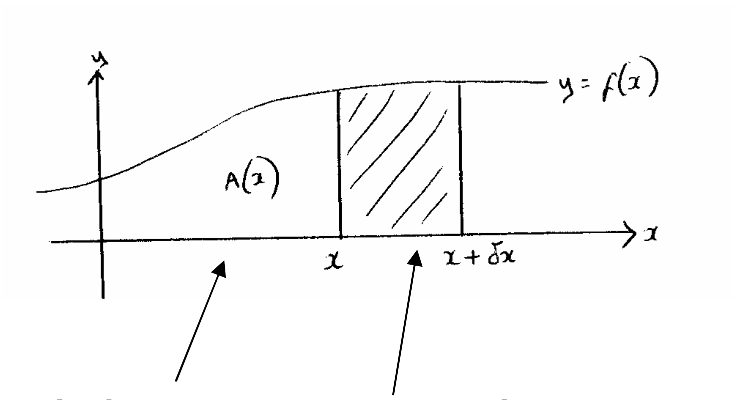
To find the area between a curve, the x-axis and the lines $x{=}a$ and $x{=}b$ you have

$$\text{Area} = \int_b^a y\ dx$$

where $y = f(x)$ is the equation of the curve



Why?



This is the area under the curve to the left of x

As x increases, the area increases

If you look at a small increase of x (δx) then the area of the increase would be $A(x + \delta x) - A(\delta x)$.
Because the area is very close to being a rectangle, then the area would be $y \times \delta y$,
as δx becomes smaller, the error of the area of the increase would be smaller also.

So $\delta A \cong y\,dx$

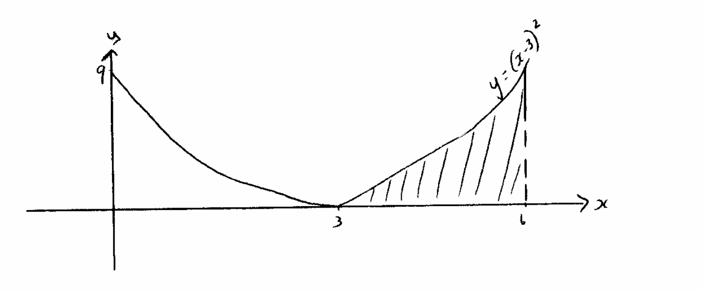
$$\frac{\delta A}{\delta x} \cong y$$ (small changes in Area is approximatley equal to $y \times$ small change in x)

As the limit of δx approaches 0 we can say that $\dfrac{da}{dx} = y$

$\therefore$ to find A you need to integrate y with respect to x

hence $A = \int y\ dx$

Example 1. Find the area under the curve $y = (x - 3)^2$ between $x = 3$ and $x = 6$.



$$= \int_3^6 (x - 3)^2 \; dx$$

$$= \int_3^6 x^2 - 6x + 9 \; dx$$

$$= \left[\frac{x^3}{3} - 3x^2 + 9x \right]_3^6$$

$$= \left(\frac{216}{3} - 3 \times 6^2 + 9 \times 6 \right) - \left(\frac{27}{3} - 3 \times 3^2 + 9 \times 3 \right)$$

$$= (72 + 108 + 54) - (9 - 27 + 27)$$

$$= 18 - 9$$

$$= 9$$