

Solving Trigonometric Equations in Degrees

Using the relationship between Sin, Cos and Tan

1. For all values of θ (except for when $\tan \theta$ is not defined and where $\cos \theta = 0$)

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Why?

$$\tan \theta = \frac{y}{x}$$

$$\frac{y}{x} = \frac{y}{r} \times \frac{r}{x} \quad \text{as } \frac{x}{r} = \cos \theta \quad \text{and } \frac{y}{r} = \sin \theta$$

$$\therefore \frac{y}{x} = \sin \theta \times \frac{1}{\cos \theta}$$

$$\therefore \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

$$\text{hence } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

2. For all values of θ , $\cos^2 \theta + \sin^2 \theta = 1$

Why?

$$\sin \theta = \frac{y}{r} \quad \text{so } y = r \sin \theta$$

$$\cos \theta = \frac{x}{r} \quad \text{so } x = r \cos \theta$$

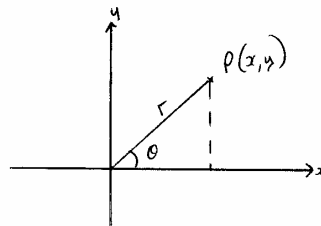
Using Pythagoras

$$x^2 + y^2 = r^2$$

$$(r \cos \theta)^2 + (r \sin \theta)^2 = r^2$$

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = r^2$$

$$\cos^2 \theta + \sin^2 \theta = 1$$



Example 1:

Simplify $\sin^2 2\theta + \cos^2 2\theta$

as $\sin^2 \theta + \cos^2 \theta = 1$

$$\sin^2 2\theta + \cos^2 2\theta = 1$$

Example 2: Show that $\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = 1 - \tan^2 \theta$

$$\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = \frac{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)}{\cos^2 \theta} \leftarrow \begin{array}{l} \text{Use} \\ \text{difference} \\ \text{of 2 squares} \end{array}$$

$$\begin{aligned} \text{as } \sin^2 \theta + \cos^2 \theta &= 1 &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \\ & &= 1 - \frac{\sin^2 \theta}{\cos^2 \theta} \end{aligned}$$

$$\text{as } \frac{\sin \theta}{\cos \theta} = \tan \theta \qquad = 1 - \tan^2 \theta$$

Example 3: Given $\cos \theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin \theta$ and $\tan \theta$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{as } \cos \theta = -\frac{3}{5} \qquad \text{so } \cos^2 \theta = \left(-\frac{3}{5}\right)^2$$

$$\therefore \sin^2 \theta + \left(-\frac{3}{5}\right)^2 = 1$$

$$\sin^2 \theta = \frac{16}{25}$$

$$\sin \theta = \frac{4}{5}$$

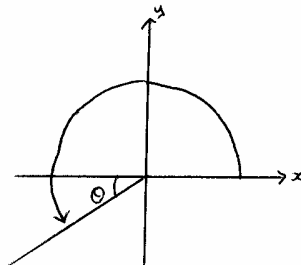
As \cos was negative in the question and reflex, it must be in quadrant 3

$$\sin \theta = -\frac{4}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{-0.8}{-0.6}$$

$$= \frac{4}{3}$$



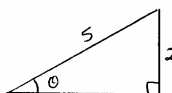
Example 4:

Given that $\sin \theta = \frac{2}{5}$ and it is obtuse. Find the exact value of $\cos \theta$

$$x = \sqrt{5^2 - 2^2}$$

$$x = \sqrt{21}$$

$$\therefore \cos \theta = \frac{\sqrt{21}}{5}$$

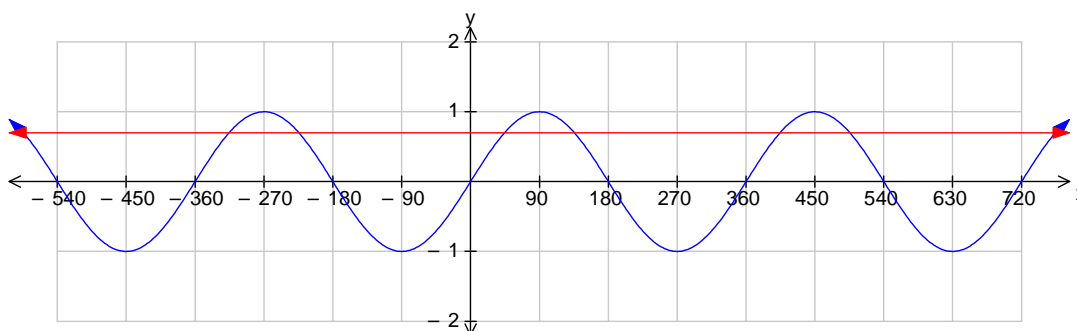


As $\sin \theta$ is positive and obtuse it must be in the 2nd quadrant.

$$\therefore \cos \theta = -\frac{\sqrt{21}}{5} \text{ because cos is negative in the 2nd quadrant}$$

Solving Simple Trigonometric Equations

If you have an equation like $\sin x = 0.7$ we can show this on a graph



Where the 2 lines intersect are the answers.

Or we can use algebra and our knowledge of trigonometric functions and quadrants.

Angle	positive	negative	How to calculate answer
0 - 90°	All	None	Keep angle
90° - 180°	Sin	Tan, cos	180° - angle
180° - 270°	Tan	Sin, cos	180° + angle
270° - 360°	Cos	Sin, tan	360° - °angle

Example 1:

Find the solution to $\sin \theta = 0.6$ for $0 \leq \theta \leq 360^\circ$

$$\sin \theta = 0.6$$

$$\theta = 36.9^\circ \text{ (1dp)}$$

$\sin \theta$ is positive in quadrants 1 and 2

$$Q1 = 36.9^\circ$$

$$Q2 = 180 - 36.9$$

$$= 143.1^\circ$$

\therefore The solutions are 36.9° & 143.1°

Example 2:

Find the solution to $\tan \theta = -\sqrt{3}$ for $0 \leq \theta \leq 360^\circ$

$$\tan \theta = \sqrt{3} \quad \text{ignore the signs to find the initial angle } \theta$$

$$\theta = 60^\circ$$

\tan is negative in quadrants 2 and 4

$$\text{Q2} = 180 - 60$$

$$= 120^\circ$$

$$\text{Q4} = 360 - 60$$

$$= 300^\circ$$

The solutions are 120° & 300°

If you need to find more solutions because one of your initial answers lie outside your region, then you can plus or minus the period of the graph.

What's that? $\cos \theta \quad \theta = 1\theta \quad \text{so } \frac{360}{1} = 360^\circ$

The period is 360° so you can either add or subtract 360° to find the other solutions

$$\sin 3\theta \quad \text{so } \frac{360}{3} = 120^\circ$$

The 3 means it has been squashed horizontally

The period here is 120° so you can either add or subtract 120° to find other solutions

Solving Trigonometric Equations in Radians

When a question is in radians then you deal with them in exactly the same way as degrees but you need to remember to change the mode on your calculator.

Example 1: Find the solution to $7\sin \theta = 4$ for $0 \leq \theta \leq 2\pi$

$$7\sin \theta = 4$$

$$\sin \theta = \frac{4}{7}$$

$$\theta = 0.608 \text{ rads (3dp)}$$

$\sin \theta$ is positive in quadrants 1 and 2

$$Q1 = 0.608 \text{ rads}$$

$$Q2 = \pi - 0.608$$

$$= 2.543 \text{ rads}$$

\therefore The solutions are 0.608 rads and 2.534 rads

Solving Harder Trigonometric Equations in Degrees

The key to solving these equations is to try and rearrange the original equation into the form $\sin \theta = C$, this will then give you the value of θ . If you have $\sin(\theta + C) = k$ then you leave the brackets till the end

Example 1: Find the solution to $3\cos(x - 30^\circ) = 1$ for $0 \leq \theta \leq 360^\circ$

$$3 \cos(x - 30) = 1$$

$$\cos(x - 30) = \frac{1}{3}$$

$$(x - 30) = 70.53^\circ$$

$$\therefore \theta = 70.53$$

cos is positive in quadrants 1 and 4

$$\text{Q1 } x - 30 = 70.53$$

$$x = 100.53^\circ$$

$$\text{Q2 } x - 30 = 360 - 70.53$$

$$x - 30 = 289.47$$

$$x = 319.47^\circ$$

\therefore The solutions are 100.53° & 319.47°

Remember 'inside the bracket leave till the end'

Solving Harder Trigonometric Equations in Radians

You deal with these in the way as you did for degrees but you must make sure you have your calculator on the right mode.

Example 1: Find the solution to $4 \sin(x - 2) = 2$ for $0 \leq x \leq 2\pi$

$$4 \sin(x - 2) = 2$$

$$\sin(x - 2) = \frac{1}{2}$$

$$x - 2 = 0.5236$$

$$\theta = 0.5236$$

sin is positive in quadrants 1 and 2

$$\text{Q1} \quad x - 2 = 0.5236$$

$$x = 2.534 \text{ rads}$$

$$\text{Q4} \quad x - 2 = 2\pi - 0.5236$$

$$x - 2 = 5.7596$$

$$x = 7.7596 \text{ rads}$$

Here we can see that the second answer is outside our range so we need to find the next solution by subtracting 2π

$$7.7596 - 2\pi = 1.4764$$

\therefore The solutions are 1.4764 rads & 2.534 rads

Solving Trigonometric Equations With More Than 2 solutions

These are equations where you no longer have a single x term but one where there is a coefficient in front of the x . Again these equations are dealt with like the brackets, we make sure we deal with the x term last, but it does bring up a different problem which is related to the number of solutions that the equation will generate.

Example 1: Find the solution to $4 \cos 2x - 1 = 2$ for $0 \leq x \leq 2\pi$

$$4 \cos 2x - 1 = 2$$

$$4 \cos 2x = 3$$

$$\cos 2x = \frac{3}{4}$$

$$2x = 0.7227 \text{ rads}$$

cos is positive in quadrants 1 and 4

$$\text{Q1} \quad 2x = 0.7227$$

$$x = 0.3614 \text{ rads}$$

$$\text{Q4} \quad 2x = 2\pi - 0.7227$$

$$2x = 5.5605$$

$$x = 2.7802 \text{ rads}$$

This is not the final answer because the period is π

$$\cos 2x \quad \text{period} = \frac{2\pi}{2} = \pi$$

The number in front of the x also tells us how many solutions there will be within any 2π or 360° range

so for $\cos 2x$ we would be looking for 4 solutions

$$0.3614 + \pi = 3.50299$$

$$2.7802 + \pi = 5.9218$$

\therefore The solutions are 0.3614, 2.7802, 3.50299, 5.9218

Always check for how many solutions you need (remember this is for a 2π or 360° range)

$$\sin 3x = 6 \text{ solutions}$$

$$\cos 4x = 8 \text{ solutions}$$

$$\cos \frac{1}{2}x = 1 \text{ solutions}$$

Solving Harder Trigonometric Equations in Radians

You need to solve them the same as any other quadratic equation

Example 1: $2 \cos^2 \theta - \cos \theta = 1$ solve for $0 \leq \theta \leq 360^\circ$

$$2 \cos^2 \theta - \cos \theta = 1$$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$\cos \theta = -\frac{1}{2} \quad \text{or} \quad \cos \theta = 1$$

$$\text{using } \cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\text{using } \cos \theta = 1$$

$$\theta = 0$$

$\cos \theta$ is negative in quadrants 2 & 3

$\cos \theta$ is positive in quadrants 1 & 4

$$\text{Q2 } \theta = 180 - 60$$

$$\theta = 120^\circ$$

$$\text{Q3 } \theta = 180 + 60$$

$$\theta = 240^\circ$$

$$\text{Q1 } \theta = 0^\circ$$

$$\text{Q4 } \theta = 360 - 0$$

$$\theta = 360^\circ$$

The solutions are $0^\circ, 120^\circ, 240^\circ, 360^\circ$

Be aware you may need to use $\sin^2 x + \cos^2 x = 1$ to simplify before you can factorise.