

Dynamics

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Vertical Motion

If a particle is falling in the earth's atmosphere then it will accelerate at 9.8ms^{-2} . By Newton's first law the particle must be experiencing a force and the only force present is the weight, so by considering the equation of motion:

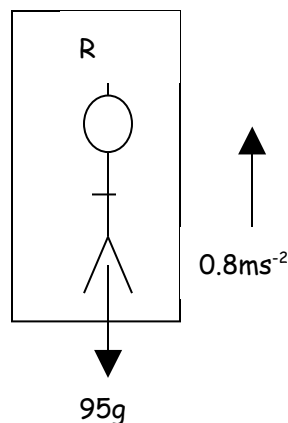
$$F = ma$$

$$\text{Weight} = mg$$

Example 3

A man of mass 95kg is traveling up in a lift. Given that the acceleration of the lift is 0.8ms^{-2} , find the force exerted on the man by the floor of the lift.

Calculate the same force when the lift is descending with the same acceleration.



Taking up to be positive and setting up an equation of motion:

$$F = ma$$

$$R - 95g = 95a$$

$$R - 931 = 95 \times 0.8$$

$$R = 1007\text{N}$$

When the lift is traveling in the opposite direction let **down** be positive so the equation of motion becomes:

$$95g - R = 95a$$

$$931 - R = 95 \times 0.8$$

$$R = 855\text{N}$$

It is worth pointing out that the reaction from the floor is less than the weight of the man when the lift is descending (and vice versa).

The next example introduces constant acceleration equations into the problem.

Example 4

A ball of mass 0.9 kg falls from a height of 22m above horizontal ground. The ball reaches the ground after t seconds. The ball sinks into the ground a distance of 1.9cm before coming to rest. The ground is assumed to exert a constant resistive force of magnitude F newtons. Find:

- the value of t to 3 sig fig;
- the value of F to 3 sig fig.

a) Using constant acceleration equations to find t :

$$s = 22, \quad a = 9.8, \quad u = 0, \quad t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$22 = 0 + 0.5 \times 9.8 \times t^2$$

$$t = 2.12 \text{ sec}$$

b) The ball is brought to rest in 1.9cm so by using constant acceleration equations we can find the deceleration and hence the force:

Firstly we need to find the speed with which the ball hits the surface

$$u = 0, \quad v = ?, \quad t = 2.12, \quad a = 9.8$$

$$v = u + at$$

$$v = 9.8 \times 2.12$$

$$v = 20.776 \text{ ms}^{-1}$$

And now for the deceleration:

$$s = 0.019, \quad a = ?, \quad v = 0, \quad u = 20.776$$

$$v^2 = u^2 + 2as$$

$$0 = 20.776^2 + 2 \times a \times 0.019$$

$$a = -11359 \text{ms}^{-2}$$

And finally using an equation of motion for the particle:

$$F = ma$$

$$F = 0.9 \times 11359$$

$$F = 10.2 \text{KN}$$

Note that we were asked for the magnitude hence the positive answer. How realistic is this answer? Ask a physics teacher for some other examples.

Example 5

A ball of mass 5kg falls from a height of 6m into a jar containing a viscous liquid. The upward force exerted by the liquid is of magnitude 75N. How far will the ball sink into the liquid? Calculate the total time that the ball is in motion.

Firstly we need to calculate the speed with which the ball hits the liquid.

Assuming that down is positive:

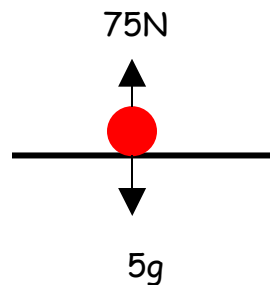
$$s = 6, \quad a = 9.8, \quad u = 0, \quad v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.8 \times 6$$

$$v = 10.84 \text{ms}^{-1}$$

Secondly we can set up an equation of motion for the ball to work out the deceleration.



$$F = ma$$

$$5g - 75 = 5a$$

$$a = -5.2 \text{ms}^{-2}$$

Thirdly we need to calculate the distance that the ball travels through the liquid before coming to rest.

$$u = 10.84, \quad v = 0, \quad a = -5.2 \quad s = ?$$

$$v^2 = u^2 + 2as$$

$$0 = 117.6 - 2 \times 5.2 \times s$$

$$s = 11.31 \text{m}$$

Finally the total time taken by the ball in motion must be done in two parts seeing as before the ball hits the liquid it has an acceleration of 9.8ms^{-2} whereas, whilst falling through the liquid, it has an acceleration of -5.2ms^{-2} .

Time to meet the surface of the liquid:

$$a = 9.8, \quad u = 0, \quad v = 10.84, \quad t = ?$$

$$v = u + at$$

$$10.84 = 0 + 9.8t$$

$$t = 1.1 \text{ sec}$$

Time to come to rest:

$$a = -5.2, \quad u = 10.84, \quad v = 0, \quad t = ?$$

$$v = u + at$$

$$0 = 10.84 - 5.2t$$

$$t = 2.1 \text{ sec}$$

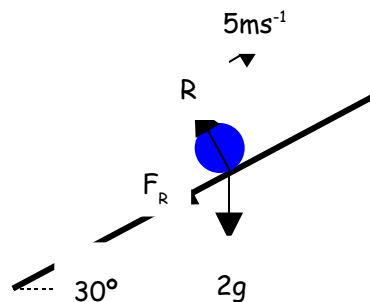
Therefore the total time in motion is 3.3 sec. The liquid is rather viscous, how realistic is the resistance force? What about a ball falling in to a jar of syrup?

The next example introduces friction on a slope. The questions are beginning to get more challenging but a good diagram is always the best place to start. Examiners regularly report that the most successful candidates in mechanics M1 and M2 always draw diagrams.

Example 6

A ball of mass 2kg is projected up a line of greatest slope inclined at an angle of 30° to the horizontal. The coefficient of friction between the plane and the ball is 0.4. The initial speed of the ball is 5 ms^{-1} . Find:

- the frictional force acting whilst the ball moves up the plane.
- the distance moved up the plane by the ball before it comes to instantaneous rest.



- Since the ball is moving then Friction must be at its maximum value.

Resolving perpendicular to the plane gives:

$$R = 2g \cos 30^\circ$$

$$R = 16.97\text{N}$$

Using $F_R = \mu R$:

$$F_R = 0.4 \times 16.97$$

$$F_R = 6.79\text{N}$$

b) The frictional force and the weight component of the ball are trying to slow the ball down. By setting up an equation of motion for the ball we can calculate the deceleration.

Assuming that uphill is positive:

$$F = ma$$

$$- F_R - \text{weight comp down the plane} = ma$$

$$-6.79 - 2g \cos 30^\circ = 2a$$

$$a = -8.295\text{ms}^{-2}$$

At the point the ball comes to instantaneous rest it will have zero velocity and we can use constant acceleration equations to calculate the distance traveled.

$$u = 5, \quad v = 0, \quad a = -8.295, \quad s = ?$$

$$v^2 = u^2 + 2as$$

$$0 = 25 - 2 \times 8.295 \times s$$

$$s = 1.51\text{m}$$