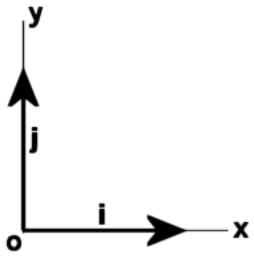
Vectors

Cartesian unit vectors and components

A **unit vector** is a vector with magnitude of 1 unit.

i, j notation



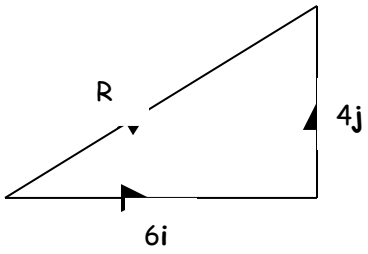
The diagram above shows the two unit vectors **i** and **j**. By definition they are vectors of magnitude one unit along the x and y coordinate axis respectively.

Example 2

A vector R = (6**i**+4**j**)
represents a displacement of:

- 6 units in the direction of the unit vector **i**,
- 4 units in the direction of the unit vector **j**.

And is displayed diagramatically as:



Adding vectors in **i**, **j** notation

When vectors are given in terms of unit vectors **i** and **j**, you can add them together by adding their terms involving **i** and **j** separately.

Example 3

Consider the two vectors a and b where: **a** = 2**i** + 3**j** **b** = 4**i** – 2**j**

a + **b** = (2**i** + 3**j**) + (4**i** – 2**j**)

=6**i** + **j**

Subtracting vectors in **i**, **j** notation

Using the same vectors a and b as in the example above

a = 2**i** + 3**j** **b** = 4**i** – 2**j**

$$a - b = (2\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - 2\mathbf{j})$$

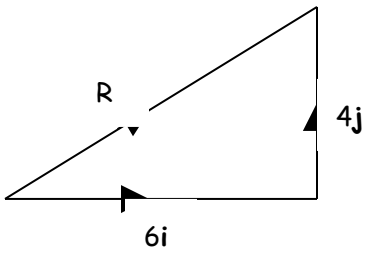
$$= (-2\mathbf{i} + 5\mathbf{j})$$

Please take care with signs as these questions are not mathematically difficult but students are sometimes prone to make silly mistakes.

Magnitude of a vector in **i**, **j** notation

When a vector R is given in terms of the unit vectors **i** and **j** you can find its magnitude by using Pythagoras' Theorem.

Using the vector R from the earlier example:



$$R^2 = \sqrt{6^2 + 4^2}$$

$$R = 10$$

The following example combines unit vectors and **i**,**j** notation.

Example 4

Find the unit vector in the direction of the vector $a = 4i + 7j$

The magnitude of a is:

$$|a| = \sqrt{4^2 + 7^2}$$

$$|a| = \sqrt{65}$$

So the unit vector in the same direction as a is

$$\frac{a}{|a|} = \frac{1}{\sqrt{65}}(4i + 7j))$$

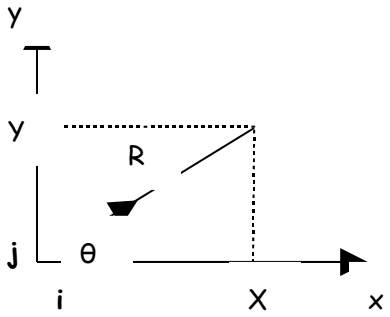
In more simple terms the vector a is $\sqrt{65}$ in length and the unit vector in the same direction of a must be $\frac{1}{\sqrt{65}}$ times the vector a.

Equal Vectors

Two vectors are equal if and only if the **i** components are equal and the **j** components are equal.

Components of a vector

Any two vectors can be written as a single vector (triangle law). However, sometimes it is useful to reverse this process. This process is called **resolving a vector into Cartesian components**. This idea is used extensively in mechanics when dealing with a number of forces (see Statics chapter).



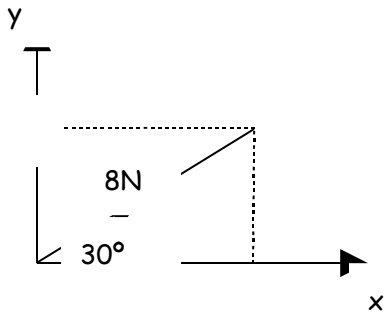
The diagram shows the vector R, which makes an angle θ with the x axis. The X and Y components can be given as:

X = R Cos θ

Y = R Sin θ

Example 5

A force of magnitude 8N acts on a particle at an angle of 30° to the horizontal. Find the x and y components of the force.



X = 8 Cos 30 = 6.93N

Y = 8 Sin 30 = 4N