

M1 January 2003

1. A railway truck P of mass 2000 kg is moving along a straight horizontal track with speed 10 m s^{-1} . The truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision Q moves with speed 5 m s^{-1} . Calculate

(a) the speed of P immediately after the collision,

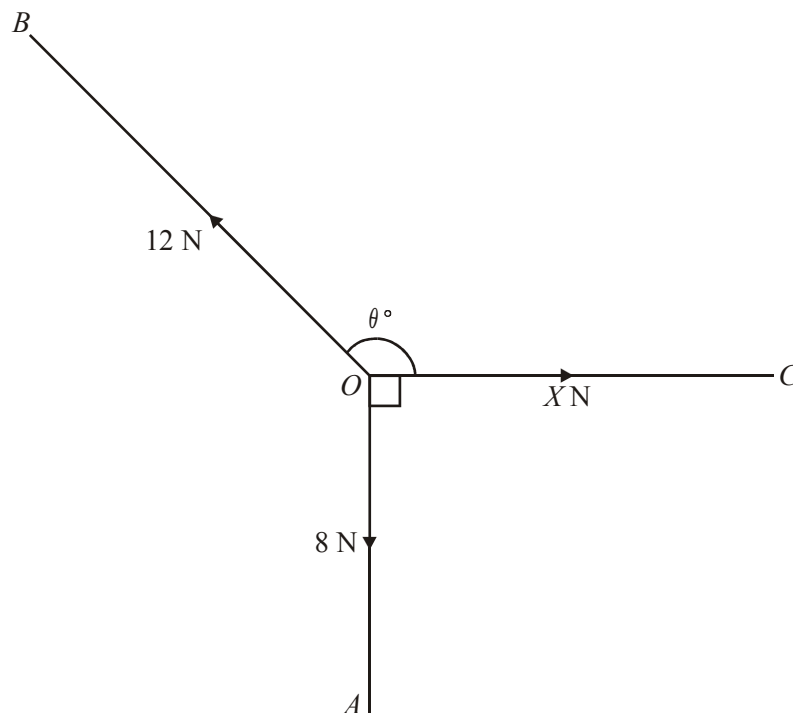
(3)

(b) the magnitude of the impulse exerted by P on Q during the collision.

(2)

(Total 5 marks)

2.



In the diagram above, $\angle AOC = 90^\circ$ and $\angle BOC = \theta^\circ$. A particle at O is in equilibrium under the action of three coplanar forces. The three forces have magnitudes 8 N , 12 N and $X \text{ N}$ and act along OA , OB and OC respectively. Calculate

(a) the value, to one decimal place, of θ ,

(3)

(b) the value, to 2 decimal places, of X .

(3)

(Total 6 marks)

3. A particle P of mass 0.4 kg is moving under the action of a constant force \mathbf{F} newtons. Initially the velocity of P is $(6\mathbf{i} - 27\mathbf{j})$ m s⁻¹ and 4 s later the velocity of P is $(-14\mathbf{i} + 21\mathbf{j})$ m s⁻¹.

(a) Find, in terms of \mathbf{i} and \mathbf{j} , the acceleration of P .

(3)

(b) Calculate the magnitude of \mathbf{F} .

(3)

(Total 6 marks)

4. Two ships P and Q are moving along straight lines with constant velocities. Initially P is at a point O and the position vector of Q relative to O is $(6\mathbf{i} + 12\mathbf{j})$ km, where \mathbf{i} and \mathbf{j} are unit vectors directed due east and due north respectively. The ship P is moving with velocity $10\mathbf{j}$ km h⁻¹ and Q is moving with velocity $(-8\mathbf{i} + 6\mathbf{j})$ km h⁻¹. At time t hours the position vectors of P and Q relative to O are \mathbf{p} km and \mathbf{q} km respectively.

(a) Find \mathbf{p} and \mathbf{q} in terms of t .

(3)

(b) Calculate the distance of Q from P when $t = 3$.

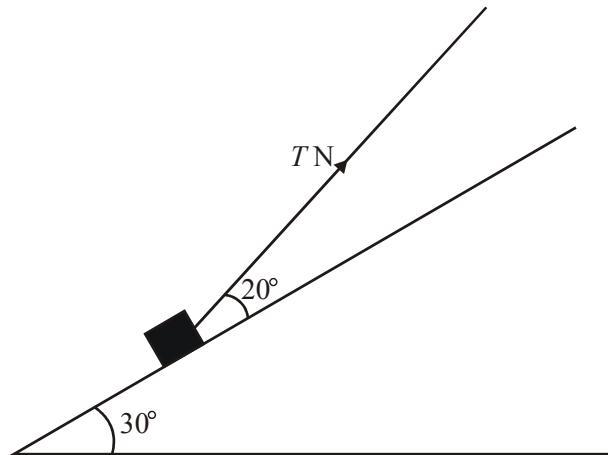
(3)

(c) Calculate the value of t when Q is due north of P .

(2)

(Total 8 marks)

5.

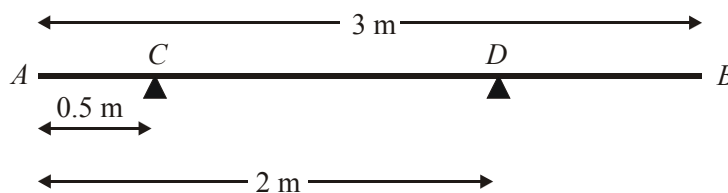


A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram above. The box is in limiting equilibrium and is about to move up the plane. The tension in the string is T newtons. The box is modelled as a particle.

Find the value of T .

(Total 10 marks)

6.



A uniform rod AB has length 3 m and weight 120 N . The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D , where $AC = 0.5\text{ m}$ and $AD = 2\text{ m}$, as shown in the diagram above. A particle of weight W newtons is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now twice the magnitude of the reaction at D .

(a) Show that $W = \frac{60}{1-x}$.

(8)

(b) Hence deduce the range of possible values of x .

(2)

(Total 10 marks)

7. A ball is projected vertically upwards with a speed $u \text{ m s}^{-1}$ from a point A which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A .

(a) Show that $u = 22.4$.

(3)

The ball reaches the ground T seconds after it has been projected from A .

(b) Find, to 2 decimal places, the value of T .

(4)

The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

(c) Find, to 3 significant figures, the value of F .

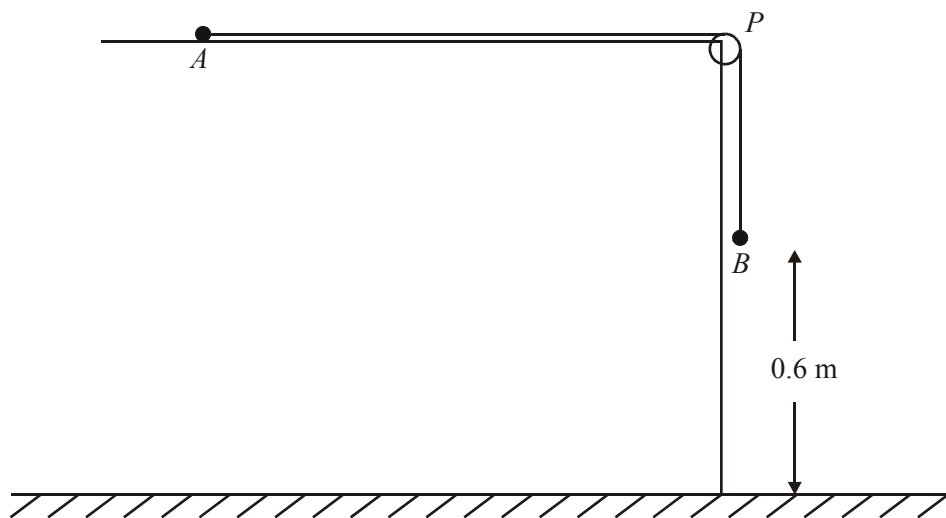
(6)

(d) State one physical factor which could be taken into account to make the model used in this question more realistic.

(1)

(Total 14 marks)

8.



A particle A of mass 0.8 kg rests on a horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle B of mass 1.2 kg which hangs freely below the pulley, as shown in the diagram above. The system is released from rest with the string taut and with B at a height of 0.6 m above the ground. In the subsequent motion A does not reach P before B reaches the ground. In an initial model of the situation, the table is assumed to be smooth. Using this model, find

- (a) the tension in the string before B reaches the ground, (5)
- (b) the time taken by B to reach the ground. (3)

In a refinement of the model, it is assumed that the table is rough and that the coefficient of friction between A and the table is $\frac{1}{5}$. Using this refined model,

- (c) find the time taken by B to reach the ground. (8)
- (Total 16 marks)**