M1 January 2003

1. A railway truck *P* of mass 2000 kg is moving along a straight horizontal track with speed 10 ms^{-1}. The truck *P* collides with a truck *Q* of mass 3000 kg, which is at rest on the same track. Immediately after the collision *Q* moves with speed 5 ms^{-1}. Calculate

(a) the speed of *P* immediately after the collision,

An easy question to start the paper. Applying conservation of momentum where u is the initial velocity and v the final velocity.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$20000 + 0 = 2000\ v_1 + 15000$$

$$5000 = 2000\ v_1$$

$$v_1 = 2.5ms^{-1}$$

(3)

(b) the magnitude of the impulse exerted by *P* on *Q* during the collision.

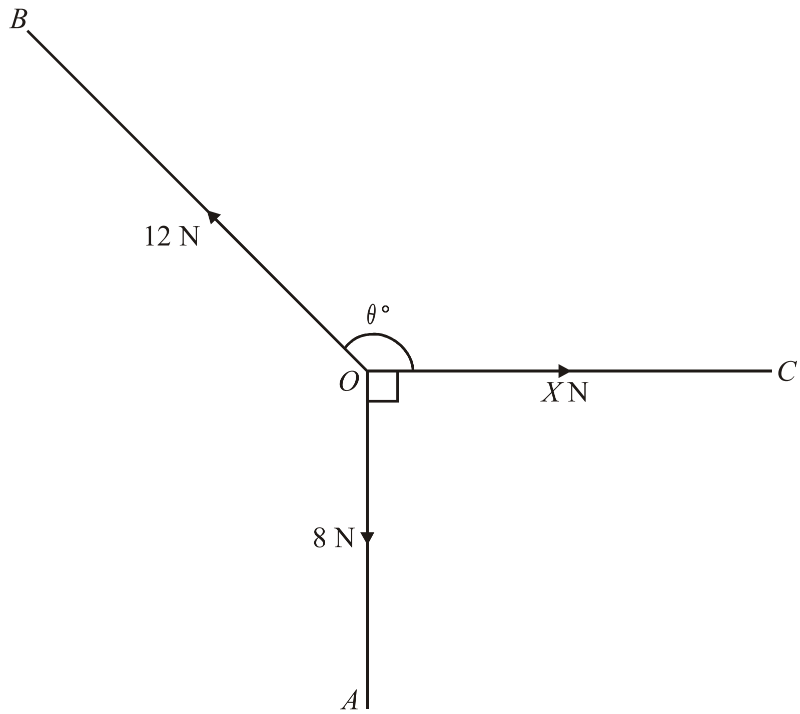
Impulse is a measure of the change in momentum:

$$= m(\ v - u\ )$$

$$= 3000 \times 5 = 15000Ns$$

(2)
(Total 5 marks)

2.



In the diagram above, angle *AOC* = 90° and angle *BOC* = θ°. A particle at *O* is in equilibrium under the action of three coplanar forces. The three forces have magnitudes 8 N, 12 N and *X* N and act along *OA*, *OB* and *OC* respectively. Calculate

(a) the value, to one decimal place, of θ°,

Resolving forces is the mainstay of mechanics. Since the object is in equilibrium the 8N force is working against the vertical component of the 12N force.

$8 = 12 \sin(180 - \theta)$

$\sin(180 - \theta) = 2/3$

$180 - \theta = 41.8°$

$\theta = 138.2°$

(3)

(b) the value, to 2 decimal places, of *X*.

Using a similar idea horizontally. The XN force is working against the horizontal component of the 12N force.

$X = 12 \times \cos 41.8°$

$X = 8.94\text{N}$

(3)

(Total 6 marks)

3. A particle *P* of mass 0.4 kg is moving under the action of a constant force **F** newtons. Initially the velocity of *P* is (6**i** – 27**j**) m s^{-1} and 4 s later the velocity of *P* is (-14**i** + 21**j**) m s^{-1}.

(a) Find, in terms of **i** and **j**, the acceleration of *P*.

Basic GCSE physics!

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

Remembering to work with the **i**'s and **j**'s separately and taking care with signs:

$$\text{Acceleration} = \frac{(-20i+48j)}{4}$$

$$= (-5i + 12j)ms^{-2}$$

(3)

(b) Calculate the magnitude of **F**.

The equation of motion is used time and again in M1.

$$F = ma$$

$$F = 0.4 \times (-5i + 12j)$$

$$F = (-2\mathbf{i} + 4.8\mathbf{j})N$$

The question asks for the magnitude. Therefore:

$$|F| = \sqrt{((-2)^2 + 4.8^2)} = 5.2N$$

Remember to read the question carefully as you could lose an answer mark.

(3)

(Total 6 marks)

4. Two ships *P* and *Q* are moving along straight lines with constant velocities. Initially *P* is at a point *O* and the position vector of *Q* relative to *O* is (6**i** + 12**j**) km, where **i** and **j** are unit vectors directed due east and due north respectively. The ship *P* is moving with velocity 10**j** km h^{-1} and *Q* is moving with velocity (-8**i** + 6**j**) km h^{-1}. At time *t* hours the position vectors of *P* and *Q* relative to *O* are **p** km and **q** km respectively.

(a) Find **p** and **q** in terms of *t*.

The position vector at time t is given by the general formula:

r = initial position + (velocity vector × time)

Therefore

$r_p = 0 + 10t\mathbf{j} = 10t\mathbf{j}$

$r_q = (6\mathbf{i} + 12\mathbf{j}) + t(-8\mathbf{i} + 6\mathbf{j}) = (6 - 8t)\mathbf{i} + (12 + 6t)\mathbf{j}$

(3)

(b) Calculate the distance of *Q* from *P* when *t* = 3.

Position of P is:

$r_p = 30\mathbf{j}$

Position of Q is:

$r_q = (6 - 24)\mathbf{i} + (12 + 18)\mathbf{j}$

$r_q = -18\mathbf{i} + 30\mathbf{j}$

Subtracting the two gives a distance of 18km.

(3)

(c) Calculate the value of *t* when *Q* is due north of *P*.

If Q is due north of P then they must have the same **i** component. P has zero **i** component looking at Q:

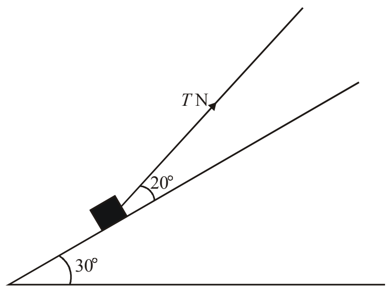
$(6 - 8t) = 0$

$t = 0.75$

(2)

(Total 8 marks)

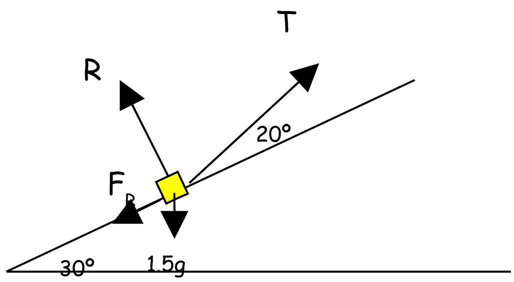
5.



A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram above. The box is in limiting equilibrium and is about to move up the plane. The tension in the string is *T* newtons. The box is modelled as a particle.

Find the value of *T*.

This question is a little difficult as the tension force is not parallel to the plane. Therefore only the component of the force that is parallel to the plane is actually pulling it along the slope. Always draw a diagram and add all forces.



Resolving forces parallel to the plane and equating to zero since the system is in equilibrium:

$$T \cos 20° - F_R - 1.5g \sin 30° = 0$$

$$F_R = 0.940T - 7.35$$

Resolving perpendicular to the plane and remembering that the perpendicular component of the tension is working with the normal reaction to oppose the weight.

$R + T \sin 20° = 1.5g \cos 30°$

$R = 12.72 - 0.342T$

The system is in equilibrium therefore:

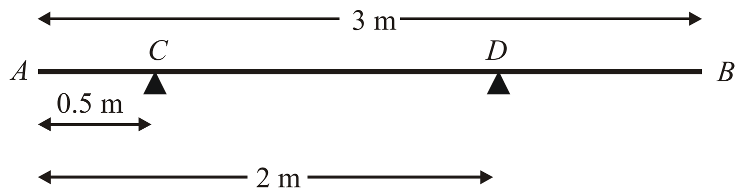
$F_R = \mu R$

$0.940T - 7.35 = 4.24 - 0.114T$

$T = 11.0N$

(Total 10 marks)

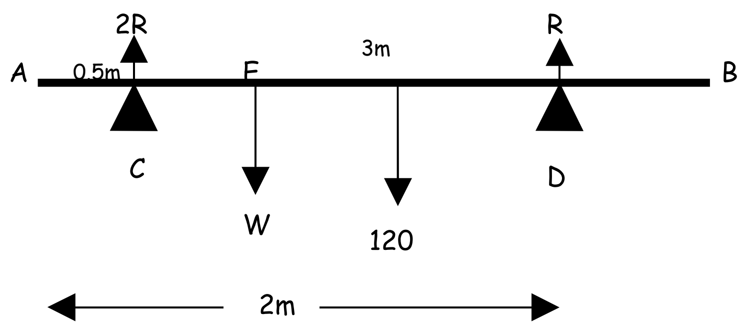
6.



A uniform rod *AB* has length 3 m and weight 120 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points *C* and *D*, where *AC* = 0.5 m and *AD* = 2 m, as shown in the diagram above. A particle of weight *W* newtons is attached to the rod at a point *E* where *AE* = *x* metres. The rod remains in equilibrium and the magnitude of the reaction at *C* is now twice the magnitude of the reaction at *D*.

(a) Show that $W = \dfrac{60}{1 - x}$.

A common trick in this type of question is to give the weight and not the mass. Take care when adding the forces to the diagram.



Taking moments about *C*:

$$W(x - 0.5) + 120 = 1.5R \qquad (1)$$

Resolving vertically gives:

$$3R = 120 + W \qquad (2)$$

Equation (2) = 2 × Equation (1)

$$2W(x - 0.5) + 240 = 120 + W$$

$$2Wx - W - W = -120$$

$$2W(x - 1) = -120$$

$$W = \frac{60}{(1 - x)}$$

(8)

(b) Hence deduce the range of possible values of *x*.

Simpler than you at first thought!

Obviously W is positive therefore:

$W > 0$, hence $x < 1$.

(2)
(Total 10 marks)

7. A ball is projected vertically upwards with a speed *u* m s^{-1} from a point *A* which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above *A*.

(a) Show that $u = 22.4$.

Using $v^2 = u^2 + 2as$

At the max height v = 0, u = ?, a = -9.8 and s = 25.6

$$v^2 = u^2 + 2as$$

$$0 = u^2 - 2 \times 9.8 \times 25.6$$

$$u = 22.4$$

(3)

The ball reaches the ground *T* seconds after it has been projected from *A*.

(b) Find, to 2 decimal places, the value of *T*.

There is no need to calculate the time to the top and then the time to fall to the floor. The question simply involves a displacement of -1.5m.

Using $s = ut + \frac{1}{2}at^2$

s = -1.5, a = -9.8, u = 22.4, t = ?

$$s = ut + \frac{1}{2}at^2$$

$$-1.5 = 22.4t - 4.9t^2$$

Solving by quadratic formula:

$$t = 4.64\,\text{sec}$$

(4)

The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude *F* newtons.

(c) Find, to 3 significant figures, the value of *F*.

Since the ball has been in motion for 4.64 sec it's velocity will have changed (a = -9.8)

$v = u + at$

$v = 22.4 - 9.8 \times 4.64$

$v = -23.07$

The ball comes to rest in 2.5cm. Using $v^2 = u^2 + 2as$ to find the deceleration. Taking down as positive:

v = 0, u = 23.07, s = 0.025, a = ?

$v^2 = u^2 + 2as$

$0 = (23.07)^2 + 2 \times 0.025 \times a$

$a = -10644.5 ms^{-2}$

After finally calculating the deceleration we can set up an equation of motion for the ball as it hits the floor. The resistive force is working against the weight of the ball, therefore:

$0.6g - F = -0.6 \times 10644.5$

$F = -6390N$ Once again asked for magnitude

$F = 6390N$

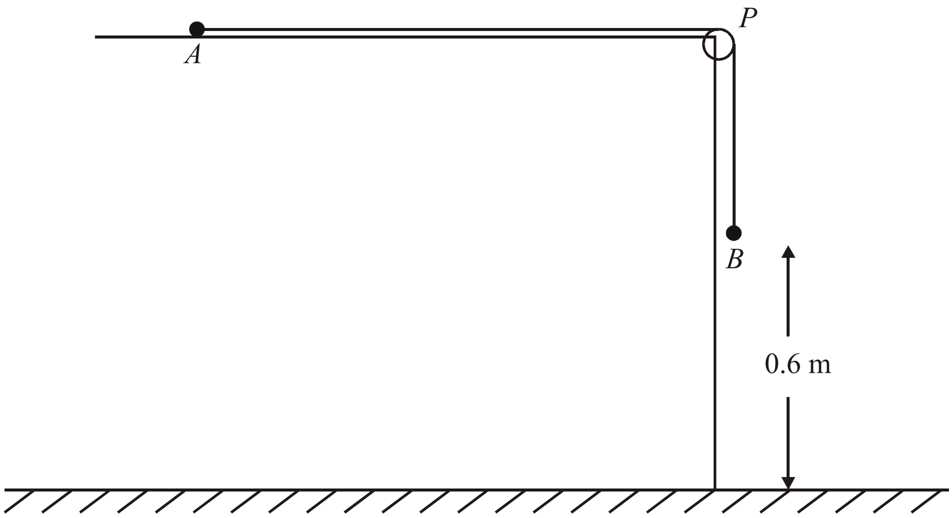
(6)

(d) State one physical factor which could be taken into account to make the model used in this question more realistic.

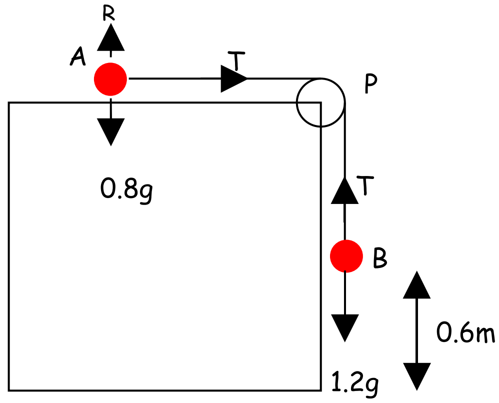
Air resistance

(1)
(Total 14 marks)

8.



A particle *A* of mass 0.8 kg rests on a horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley *P* fixed at the edge of the table. The other end of the string is attached to a particle *B* of mass 1.2 kg which hangs freely below the pulley, as shown in the diagram above. The system is released from rest with the string taut and with *B* at a height of 0.6 m above the ground. In the subsequent motion *A* does not reach *P* before *B* reaches the ground. In an initial model of the situation, the table is assumed to be smooth. Using this model, find



(a) the tension in the string before *B* reaches the ground,

Setting up equations of motion for the two particles:

For *A* $T = 0.8a$ For B $1.2g - T = 1.2a$

Adding the two equations to eliminate T:

$$1.2g = 2a$$

$$a = 0.6g = 5.88ms^{-2}$$

Therefore $T = 0.8 \times 5.88 = 4.70N$

(5)

(b) the time taken by *B* to reach the ground.

Using $s = ut + \frac{1}{2}at^2$

$s = 0.6$, $u = 0$, $a = 5.88$

$$s = ut + \frac{1}{2}at^2$$

$$0.6 = 0.5 \times 5.88 \times t^2$$

$$t = 0.452 sec$$

(3)

In a refinement of the model, it is assumed that the table is rough and that the coefficient of friction between *A* and the table is $\frac{1}{5}$. Using this refined model,

(c) find the time taken by *B* to reach the ground.

The equation of motion for *A* will have to change to account for the fact that friction will be slowing the particles down.

For *A* $T - F_R = 0.8a$ For B $1.2g - T = 1.2a$

Resolving vertically for *A* gives:

$$R = 0.8g$$

Therefore $F_R = 0.16g$

and the equation of motion for A becomes:

$$T - 0.16g = 0.8a$$

Adding the two new equations together gives:

$$1.04g = 2a$$

$$a = 5.096ms^{-2}$$

Particle B falls a distance of 0.6m with the acceleration calculated above.

Using

$s = 0.6$, $u = 0$, $a = 5.096$

$$s = ut + \frac{1}{2}at^2$$

$$0.6 = 0.5 \times 5.096 \times t^2$$

$$t = 0.485 sec$$

(8)
(Total 16 marks)