

Kinematics

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Kinematics is the study of the motion of particles. In M1 all motion will have constant acceleration. This leads to the development of the constant acceleration equations.

Definitions

a = acceleration (ms^{-2})

u = initial velocity (ms^{-1})

v = final velocity (ms^{-1})

t = time (second)

s = displacement (metres)

If we know any of the three we can find the other two.

From GCSE you should remember that:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$\therefore a = \frac{v-u}{t}$$

$$\text{so } v = u + at \quad (1)$$

As mentioned above the acceleration is constant hence the average velocity is simply the average of u and v.

$$\text{average velocity} = \frac{u+v}{2}$$

Another definition is that:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{s}{t}$$

Therefore:

$$\frac{u + v}{2} = \frac{s}{t}$$

$$\therefore s = \left(\frac{u + v}{2} \right) t \quad (2)$$

By using equation (1) we can eliminate v.

$$\therefore s = \left(\frac{u + u + at}{2} \right) t$$

$$s = ut + \frac{1}{2} at^2 \quad (3)$$

If equation (1) is rearranged to make t the subject:

$$t = \frac{v - u}{a}$$

Then by substituting into equation (2):

$$\therefore s = \left(\frac{u + v}{2} \right) \left(\frac{v - u}{a} \right)$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \quad (4)$$

You must learn all four equations above and remember that they only apply to constant acceleration problems. Since velocity is a vector quantity, getting the direction right in these problems is vital.

Example 1

A particle is moving in a straight line from O to P with a constant acceleration of 4ms^{-2} . Its velocity at P is 48ms^{-1} and it takes 12 seconds to travel from O to P. Find (a) the particle's velocity at O and (b) the distance OP.

With all problems of this nature, write down what you are given and the one required. It should then be obvious as to which equation you need to use.

$$a) \quad a = 4, \quad u = ?, \quad v = 48, \quad t = 12$$

Obviously we need to use equation (1):

$$v = u + at$$

$$48 = u + 4 \times 12$$

Therefore $u = 0 \text{ms}^{-1}$

$$b) \quad a = 4, \quad u = 0, \quad v = 48, \quad t = 12, \quad s = ?$$

Using equation (3):

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times 4 \times 12^2$$

$$s = 288\text{m}$$

Example 2

Percy starts from rest at a station S and moves with constant acceleration. He passes a signal box B 14 seconds later with a speed of 84kmh^{-1} . Modelling the train as a particle, find the acceleration of the train in ms^{-2} and the distance in metres between the station and the signal box.

$$u = 0, \quad t = 14, \quad v = 84\text{kmh}^{-1} \quad a = ?, \quad s = ?$$

Using equation (1) :

$$v = u + at$$

$$84000/60/60 = 0 + 14a$$

$$a = 1.67\text{ms}^{-2}$$

Once again using equation (3)

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times \frac{5}{3} \times 14^2$$

$$s = 163.33\text{m} = 163\text{m}$$

Example 3

ET is travelling up a hill on his BMX. He experiences a constant retardation of magnitude 3ms^{-2} . Given that his speed at the bottom of the hill was 15ms^{-1} determine how far he will travel before he comes to rest.

Retardation is simply deceleration.

$$a = -3, \quad u = 15, \quad v = 0, \quad s = ?$$

There is no mention of time therefore we use equation (4):

$$v^2 = u^2 + 2as$$

$$0 = 225 - 2 \times 3 \times s$$

$$s = 37.5\text{m}$$