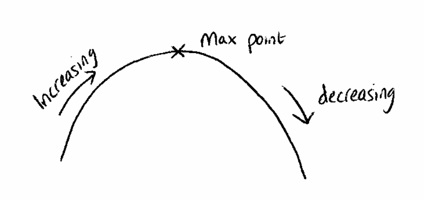
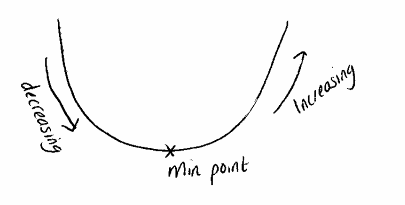
Finding Coordinates of a Stationary Point

When f(x) stops increasing and begins to decrease it is called a maximum point.



When f(x) stops decreasing and begins to increase it is called a minimum point.



The maximum and minimum points are called turning points.
At these points the gradient is zero.
That means that $f^1(x) = 0$

Example 1. Find the coordinate of the turning point of the function $f(x) = 5x^2 + 3x + 2$.
State whether it is a maximum or minimum turning point.

$f(x) = 5x^2 + 3x + 2$

$f^1(x) = 10x + 3$

$f^1(x) = 0$ for a stationary point

$\therefore \quad 10x + 3 = 0$

$10x = -3$

$x = -\dfrac{3}{10}$

when $x = -0{\cdot}3 \qquad y = 5x^2 + 3x + 2$

$y = 5 \times (-0{\cdot}3)^2 + (3 \times -0{\cdot}3) + 2$

$y = 0{\cdot}45 - 0{\cdot}9 + 2$

$y = 1{\cdot}55$

$\therefore$ the turning point is $(-0.3, 1.55)$

when x = – 1	x = – 0.3	x = 0
$f^1(x) = -7$	$f^1(x) = 0$	$f^1(x) = 3$
\	–	/

by looking at the lines we can see that the turning point is a minimum

Another way to find out if it's a maximum or a minimum point is to calculate the 2nd derivative, as this measures the change in the gradient.

If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ its a minimum point

If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$ its a maximum point

If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ it could be a minimum,maximum or point of inflection

If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ then its a point of inflection

Example 2. Find the greatest value of $f(x) = 3 + 2x - x^2$

This means find the y coordinate of the maximum turning point

$f(x) = 3 + 2x - x^2$

$f^1(x) = 2 - 2x$

$f^1(x) = 0$

$0 = 2 - 2x$

$2x = 2$

$x = 1$

when $x = 1$ $\quad y = 3 + 2x - x^2$

$y = 3 + (2 \times 1) - (1^2)$

$y = 3 + 2 - 1$

$y = 4$

$\therefore$ Greatest value of $f(x) = 4$

Example 3. Find the coordinates of the points where the gradient is zero, state where the points are max, min or a point of inflexion, for the function $f(x) = x^3 - x^2 - x + 1$

$y = x^3 - x^2 - x + 1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\frac{dy}{dx} = 0$ for turning points

$\therefore \quad 3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = 1$ or $x = -\frac{1}{3}$

when $x = 1 \quad y = x^3 - x^2 - x + 1$

$y = 1^3 - 1^2 - 1 + 1$

$y = 0 \qquad (1,0)$

when $x = -\frac{1}{3} \quad y = x^3 - x^2 - x + 1$

$$y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 1$$

$$y = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1$$

$$y = 1\frac{5}{27} \qquad \left(-\frac{1}{3}, 1\frac{5}{27}\right)$$

considering $\frac{d^2y}{dx^2} = 6x - 2$

$at\ x = 1 \qquad \frac{d^2y}{dx^2} = 4 \qquad \therefore \frac{d^2y}{dx^2} > 0 \qquad \therefore (1,0)$ is a minimum point

$at\ x = -\frac{1}{3} \qquad \frac{d^2y}{dx^2} = -4 \qquad \therefore \frac{d^2y}{dx^2} > 0 \qquad \therefore \left(-\frac{1}{3}, 1\frac{5}{27}\right)$ is a maximum point