Measuring Angles in Radians

"A radian is the angle subtended at the centre of a circle by an arc length whose length is equal to that of the radius of the circle". This means a radian is the angle formed when the arc length and the radius are the same.

The number of radians in a circle = $\frac{\text{length of circumference}}{\text{Length of radius}}$

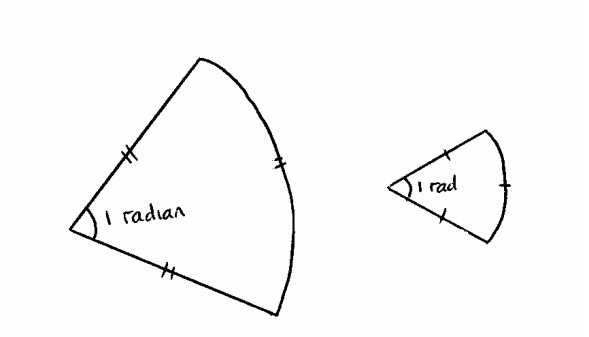
$$= \frac{2\pi r}{r}$$

$$= 2\pi$$

$\therefore$ $360° = 2\pi$ *rads*

$180° = \pi$ *rads*

1 *rad* $= \frac{180}{\pi} \approx 57{\cdot}3°$



Changing Degrees to Radians

Rule:- *Multiply by* $\frac{\pi}{180°}$

Example 1. Convert 45° to radians

$$45 \times \frac{\pi}{180} = \frac{45\pi}{180}$$

$$= \frac{\pi}{4}$$

Leave your answer in terms of π unless asked for more accuracy

Example 2. Convert 75° to radians, give your answer to 2sf.

$$75 \times \frac{\pi}{180} = \frac{75\pi}{180}$$

$$= 1{\cdot}308996$$

$$= 1{\cdot}3 \text{ (2}sf\text{)}$$

Changing Radians to Degrees

Rule: *Multiply by* $\frac{180°}{\pi}$

Example 1. *Convert* $\frac{2\pi}{3}$ *rads to degress*

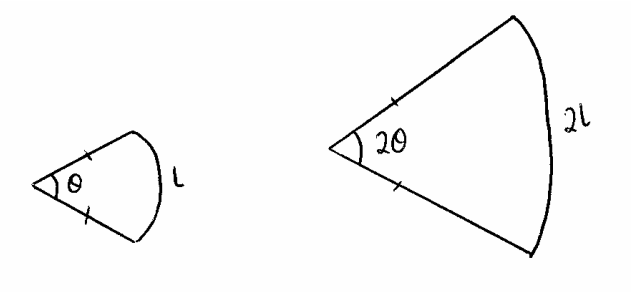
$$\frac{2\pi}{3} \times \frac{180}{\pi} = 120°$$

Example 2. *Convert* $20{\cdot}1^c$ to degrees

$$2{\cdot}1 \times \frac{180}{\pi} = 120{\cdot}3°$$

Finding Arc Lengths

The length of an arc is always proportional to the angle at the centre of the arc and the radius of the arc. So, if 2 arcs have the same radius but one has an angle twice the size of the other, it means one arc length will be twice the size of the other.



Formula for finding an arc length: – $l = r\theta$ where l = arc length,

r = radius

θ = angle at the centre

Why?

$$\frac{l}{2\pi r} = \frac{\theta}{2\pi}$$

$$(\times 2\pi r) \qquad l = \frac{2\theta\pi r}{2\pi}$$

$$l = r\theta$$

$$\frac{\text{Length of arc}}{\text{Circumference}} = \frac{\text{angle at the centre}}{\text{total angle at centre}}$$

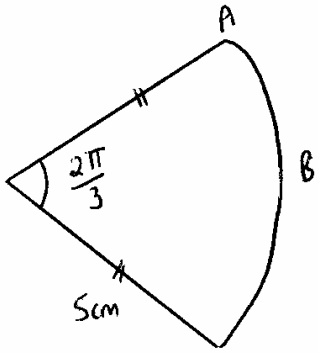
Ratio of sides = Ratio of angles

Example 1. Find the length of the arc ABC.

$l = r\theta$ $\qquad r = 5cm \;\; \theta = \frac{2\pi}{3}$

$$l = 5 \times \frac{2\pi}{3}$$

$$l = \frac{10\pi}{3} \text{ or } 10{\cdot}47cm$$



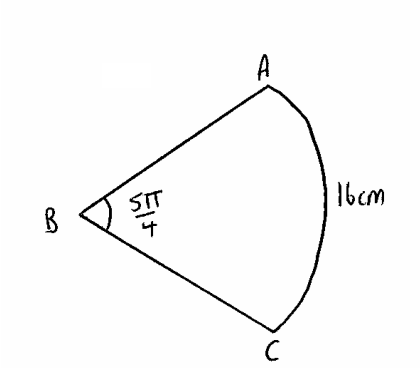
Example 2. Find the radius of the sector ABC

$l = r\theta \qquad r = 16cm \quad \theta = \frac{5\pi}{4}$

$16 = r \times \frac{5\pi}{4}$

$16 \times \frac{4}{5\pi} = r$

$\frac{64\pi}{5\pi} = r$ or $r = 4{\cdot}07cm$

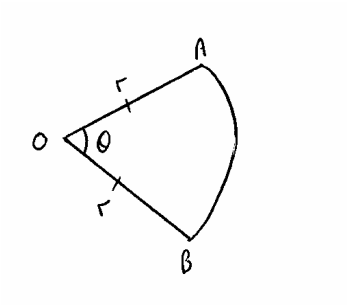


Example 3. An arc AB or a circle, with centre o and radius r cm, subtends an angle of θ radians at O. The perimeter of the sector AOB is P cm.
Express r in terms of θ.

$p = (2 \times radius) + \text{arc length}$

$p = 2r + r\theta$

$\frac{p}{2} + \theta = r$



Finding the Area of a Sector

Formula for area of a sector is $A = \frac{1}{2} r^2 \theta$ where r = *radius* , θ = angle at centre

Why?

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$(\times \pi r^2) \qquad A = \frac{\theta}{2\pi} \times \pi r^2$$

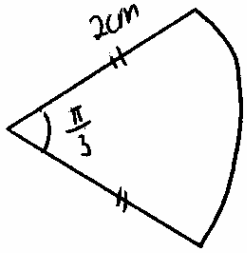
$$A = \frac{r^2\theta}{2} \quad \text{or} \quad A = \frac{1}{2} r^2 \theta$$

$$\frac{\text{Area of Sector}}{\text{Area of circle}} = \frac{\text{angle at the centre}}{\text{total angle at centre}}$$

Ratio of areas = Ratio of angles

NB. If the question gives the angle at the centre in degrees it must be changed to radians

Example 1. *Find the area of the sector ABC, where* $\angle ABC = \frac{\pi}{3}$ *and* $r = 2cm$, *give your answer in terms of* π



$$A = \frac{1}{2} r^2 \theta \qquad \text{where } r = 2\text{cm}, \ \theta = \frac{\pi}{3}$$

$$A = \frac{1}{2} \times 2 \times 2 \times \frac{\pi}{3}$$

$$A = \frac{2\pi}{3} cm^2$$

Example 2. *Find the area of the sector ABC, where* $\angle ABC = 60°$ *and* $r = 8cm$, *give your answ*

$$60° = 60 \times \frac{\pi}{180}$$

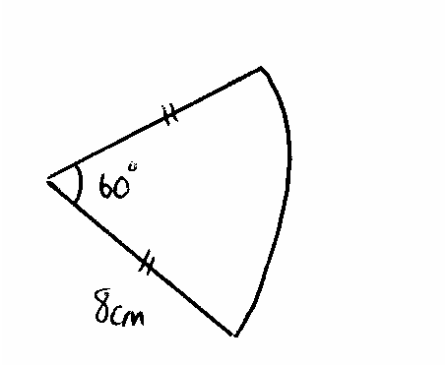
$$= \frac{\pi}{3}$$

$$A = \frac{1}{2} r^2 \theta \quad \text{where } \theta = \frac{\pi}{3} \quad , r = 8cm$$

$$A = \frac{1}{2} \times 8 \times 8 \times \frac{\pi}{3}$$

$$A = \frac{32\pi}{3}$$

$$A = 33{\cdot}5\, cm^2 \ (2sf)$$



Finding the Area of a Segment

Formula for the area of a segment:-

$$A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \, Sin\, \theta$$

or $A = \frac{1}{2} r^2 (\theta - \sin\theta)$ where r = radius, θ = angle at centre

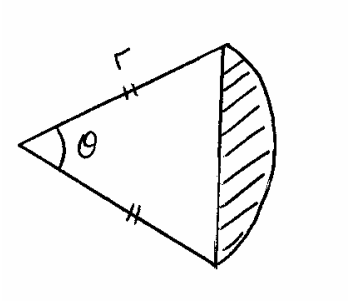
Why?

Area of segment = Area of sector – area of a triangle

$$A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta$$

as a and b = r $\therefore ab = r^2$

$$A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta$$



Example 1. Find the area of the shaded segment.

$$A = \frac{1}{2} r^2 (\theta - \sin\theta) \quad \text{where r = 9cm and } \theta = \frac{\pi}{6}$$

$$A = \frac{1}{2} \times 9^{2\times} \left(\frac{\pi}{6} - \sin\frac{\pi}{6} \right)$$

$$A = 40{\cdot}5 \times (0{\cdot}023598775)$$

$$A = 0{\cdot}95575 \; (5sf)$$