Using Factorial Notation to work out the Coefficients in the Binomial Expansion

The Binomial Expansion is:-

$$(a + b)^n = (a + b)(a + b)(a + b)........(a + b)$$

$$= ({}^nC_0)a^n b^0 + ({}^nC_1)a^{n-1}b^1 + ({}^nC_2)a^{n-2}b^2 + ...({}^nC_n)a^{n-n}b^n$$

or $$\binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 .... \binom{n}{n} a^{n-n} b^n$$

Example 1. Use the binomial theorem to find the expansion of $(3 - 2x)^5$

$$= \binom{5}{0} 3^5 (-2x)^0 + \binom{5}{1} 3^4 (-2x)^1 + \binom{5}{2} 3^3 (-2x)^2 + \binom{5}{3} 3^2 (-2x)^3$$

$$+ \binom{5}{4} 3^1 (-2x)^4 + \binom{5}{5} 3^0 (-2x)^5$$

$$= 243 - 810x + 1080x^2 - 720x^3 + 240x^4 - 32x^5$$

Example 2. a) Write down the first 4 terms of the expansion $\left(1 - \frac{6}{x}\right)^6$

$$= \binom{6}{0} (1)^6 \left(-\frac{x}{10}\right)^0 + \binom{6}{1} (1)^5 \left(-\frac{x}{10}\right)^1 + \binom{6}{2} (1)^4 \left(-\frac{x}{10}\right)^2 + \binom{6}{3} (1)^3 \left(-\frac{x}{10}\right)^3$$

$$= 1 + \left(6 \times \frac{x}{10}\right) + \left(15 \times \frac{x^2}{100}\right) + \left(20 \times -\frac{x^3}{1000}\right)$$

$$= 1 - \frac{6x}{10} + \frac{15x^2}{100} - \frac{20x^3}{1000}$$

$$= 1 - 0{\cdot}6x + 0{\cdot}15x^2 - 0{\cdot}02x^3$$

b) By substituting an appropriate value of x, find an approximate value to $(0.99)^6$. Use your calculator to find the degree of accuracy of your approximation.

This means we want $\left(1 - \frac{x}{10}\right) = 0{\cdot}99$

$$0{\cdot}01 = \frac{x}{10}$$

$$0{\cdot}1 = x$$

substitute x = 0.1 into $(1 - 0{\cdot}6x + 0{\cdot}15x^2 - 0{\cdot}02x^3)$

$$= 1 - (0{\cdot}6 \times 0{\cdot}1) + (0{\cdot}15 \times 0{\cdot}1^2) - (0{\cdot}02 \times 0{\cdot}1^3)$$

$$= 1 - 0{\cdot}06 + 0{\cdot}0015 - 0{\cdot}00002$$

$$= 0{\cdot}94148$$

Using a calculator $0{\cdot}99^6 = 0{\cdot}941480149$

so approximation is accurate to 5dp as this how far the two answers are the same