Pascal's Triangle

Pascal's Triangle looks like this

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

When we expand expression such as $(x + y)^4$ we find it follows the same pattern.

Example 1. $(x + y)^4 = x^2 + 2xy + y^2$ Notice the pattern of the coefficients (1,2,1)

Example 2. $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y$ Here the coefficients are (1,3,3,1)

The key is to notice the 3 patterns:-

1. The coefficients follow Pascal's triangle sequence
2. The powers of one decrease while the other increase
3. The total power in each expression adds up to the original power in the questions

Example 3. $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^3y^2 + 5xy^4 + y^5$

So now we know this:

$(a + b)^0 = 1$ *Line* 1

$(a + b)^1 = 1a + 1b$ *Line* 2

$(a + b)^2 = 1a^2 + 2ab + 1b^2$ *Line* 3

$(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$ *Line* 4

$(a + b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$ *Line* 5

Example 4. Expand $(2x + 3y)^3$ using Pascal's triangle

4th line is 1,3,3,1

$= 1(2x)^3(3y)^0 + 3(2x)^2(3y)^1 + 3(2x)^1(3y)^2 + 1(2x)^0(3y)^3$

$= 8x^3 + (3 \times 4x^2 \times 3y) + (3 \times 2x \times 9y^2) + (1 \times 27y^3)$

$= 8x^3 + 36x^2y + 54xy^2 + 27y^3$

Example 5. Fully expand $(1 + 3x)(1 + 2x)^3$

$$(1 + 2x)^3 = 1(1)^3(2x)^0 + 3(1)^2(2x)^1 + 3(1)^1(2x)^2 + 1(1)^0(2x)^3$$

$$= 1 + 6x + 12x^2 + 8x^3$$

$$(1 + 3x)(1 + 2x)^3 = (1 + 3x)(1 + 6x + 12x^2 + 8x^3)$$

$$= 1 + 6x + 12x^2 + 8x^3 + 3x + 18x^2 + 36x^3 + 24x^4$$

$$= 1 + 9x + 30x^2 + 44x^3 + 24x^4$$