Solving Equations in the Form $a^x = b$

As some calculator can only work in base 10, we need to make all calculations into base 10.

Example 1. Solve the following equations $2^x = 57 \cdot 35$

$$\log_{10} 2^x = \log_{10} 57 \cdot 35$$

$$x \log_{10} 2 = \log_{10} 57 \cdot 35$$

$$x = \frac{\log_{10} 57 \cdot 35}{\log_{10} 2}$$

$$x = 5 \cdot 842 \ (4sf) \text{ using a calculator}$$

Example 2. Solve the following equation $5^{x+1} = 3^{x+2}$

$$\log_{10} 5^{x+1} = \log_{10} 3^{x+2}$$

$$(x + 1)\log_{10} 5 = (x + 2) \log_{10} 3$$

$$x \log_{10} 5 + \log_{10} 5 = x \log_{10} 3 + 2 \log_{10} 3$$

$$x(\log_{10} 5 - \log_{10} 3) = \log_{10} 9 - \log_{10} 5$$

$$x = \frac{\log_{10} 9 - \log_{10} 5}{\log_{10} 5 - \log_{10} 3}$$

$$x = 1 \cdot 151 \ (4sf)$$

Example 3. Solve the equation $2^{2x} - 6(2^x) + 5 = 0$

Let $y = 2^x$ as $2^{2x} - 6(2^x) + 5 = 0$

then $y^2 - 6(y) + 5 = 0$

$$y^2 - 6y + 5 = 0$$

$$(y - 5)(y - 1) = 0$$

$$y = 5 \text{ or } y = 1$$

If $y = 5$ then $2^x = 5$

$$\log_{10} 2^x = \log_{10} 5$$

$$x \log_{10} 2 = \log_{10} 5$$

$$x = \frac{\log_{10} 5}{\log_{10} 2}$$

$$x = 2{\cdot}32 \ (3sf)$$

if y = 1 then $2^x = 1$

$$x = 0$$

$\therefore$ two solutions are 0 and 2.32