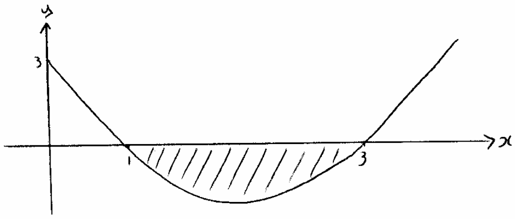
Calculating Areas of Curves Under the x-axis

Example 1. Find the area of the finite region bounded by the curve $y = x^2 - 4x + 3$ and the x-axis.



$$f(x) = x^2 - 4x + 3$$

$$y = 0 \quad (x - 1)(x - 3) = 0$$

$$x = 1 \text{ or } x = 3$$

this means it cuts the x axis at 1 and 3

$$\int_1^3 x^2 - 4x + 3 \; dx$$

$$\left[\frac{x^3}{3} - 2x^2 + 3x \right]_1^3$$

$$= \left(\frac{3^3}{3} - 2 \times 3^2 + 3 \times 3 \right) - \left(\frac{1^3}{3} - 2 \times 1^2 + 3 \times 1 \right)$$

$$= (9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3 \right)$$

$$= (0) - \left(1\frac{1}{3} \right)$$

$$= -1\frac{1}{3}$$

$$\therefore \quad \text{Area} = 1\frac{1}{3}$$

An area cannot be negative so ignore the sign

Example 2. Find the area of the finite region bounded by the curve $y = x^3 - x^2 - 6x$ and the x- axis.

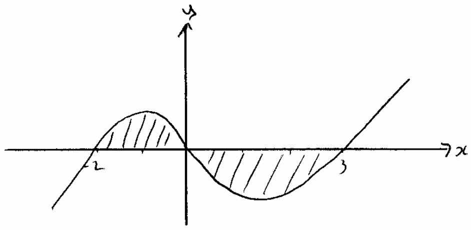
$$y = x^3 - x^2 - 6x$$

$$y = 0 \qquad 0 = x^3 - x^2 - 6x$$

$$0 = x(x^2 - x - 6)$$

$$0 = x(x + 2)(x - 3)$$

$$x = 0 \quad x = -2 \quad x = 3$$



Because there are two areas, deal with them separately

$$\int_{-2}^{0} x^3 - x^2 - 6x \; dx \qquad + \qquad \int_{0}^{3} x^3 - x^2 - 6x \; dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x \right]_{-2}^{0} \qquad \qquad = \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x \right]_{0}^{3}$$

$$= (0) - \left(4 - \frac{4}{32} + 6 \right) \qquad \qquad = \left(\frac{81}{4} - 9 - 9 \right) - (0)$$

$$= -8\frac{2}{3} \qquad \qquad = 2\frac{1}{4}$$

$$\therefore \text{ Total area } = 8\frac{2}{3} + 2\frac{1}{4}$$

$$= 10\frac{11}{12}$$