<u>Factor Theorem</u>

To find out the factors of a polynomial you can quickly substitute in values of x to see which give you a value of zero.

Example 1: Given $f(x) = x^3 + x^2 - 4x - 4$. Use the factor theorem to find a factor

$$f(x) = x^3 + x^2 - 4x - 4$$

$$f(1) = (1)^3 + (1)^2 - (4 \times 1) - 4$$

$$= -6 \qquad \therefore (x - 1) \text{ is not a factor}$$

$$f(2) = (2)^3 + (2)^2 - (4 \times 2) - 4$$

$$= 0 \qquad \therefore (x - 2) \text{ is a factor}$$

To go from the x value to the factor simply put it into a bracket and change the sign.

Remember this:-

$$(x + 3)(x - 2) = 0$$

$$x = -3 \text{ and } x = 2$$

We are now going in reverse by putting the x value back into the brackets

Example 2: Given $f(x) = 3x^2 + 8x^2 + 3x - 2$. Factorise fully the given function

$$f(x) = 3x^3 + 8x^2 + 3x - 2$$

$$f(1) = 3 \times (1)^3 + 8 \times (1)^2 + (3 \times 1) - 2$$

$$= 12 \qquad \therefore (x - 1) \text{ is not a factor}$$

$$f(2) = 3 \times (2)^3 + 8 \times (2)^2 + (3 \times 2) - 2$$

$$= 60 \qquad \therefore (x - 2) \text{ is not a factor}$$

$$f(-1) = 3 \times (-1)^3 + 8 \times (-1)^2 + (3 \times -1) - 2$$

$$= 0 \qquad \therefore (x + 1) \text{ is a factor}$$

To fully factorise it we now need to find the quotient. We do this by dividing by the factor we have just found.

$$
\begin{array}{rl}
 & \quad 3x^2 + 5x - 2 \\
= & (x + 1)\overline{\big)\,3x^3 + 8x^2 + 3x - 2} \\
 & - \underline{\;3x^3 + 3x^2\;} \\
 & \qquad\qquad 5x^2 + 3x \\
 & \qquad\quad - \underline{\;5x^2 + 5x\;} \\
 & \qquad\qquad\qquad -2x - 2 \\
 & \qquad\qquad\quad - \underline{\;-2x - 2\;} \\
 & \qquad\qquad\qquad\qquad 0
\end{array}
$$

$$\therefore (x + 1)(3x^2 + 5x - 2) = (x + 1)(3x - 1)(x + 2)$$

We have factorised the quotient further to get the fully factorised answer