Simplifying Algebraic Fractions

Algebraic fractions can be simplified by division.

Example 1: Simplify $\frac{3x^3 + 5x^2 - 2x}{x}$

$$= \frac{3x^3}{x} + \frac{5x^2}{x} - \frac{2x}{x}$$

$$= 3x^2 + 5x - 2$$

Example 2: Simplify $\frac{6x^4 - 5x^3 + 2x^2}{-2x}$

$$= \frac{6x^4}{-2x} - \frac{5x^3}{-2x} + \frac{2x^2}{-2x}$$

$$= -3x^3 + \frac{5x^2}{2} - x$$

Factorizing to Cancel

Sometimes you need to factorise before you can simplify

Example 1: Simplify $\frac{x^2 + 7x + 12}{x + 3}$

$$= \frac{(x + 3)(x + 4)}{x + 3}$$

$$= x + 4$$

Example 2: Simplify $\frac{2x^2 + 5x - 12}{2x^2 - 7x + 6}$

$$= \frac{(2x - 3)(x + 4)}{(x - 2)(2x - 3)}$$

$$= \frac{x + 4}{x - 2}$$

Dividing a Polynomial by (x±p)

You divide polynomials in the same way as you perform long division.

Example 1: Divide $6x^3 + 28x^2 - 7x + 15$ by $(x + 5)$

$$
\begin{array}{rl}
 & 6x^2 - 2x + 3 \\
= \quad (x+5) & \overline{)\,6x^3 + 28x^2 - 7x + 15} \\
- & \underline{6x^3 + 30x^2} \\
 & -2x^2 - 7x \\
- & \underline{-2x^2 - 10x} \\
 & 3x + 15 \\
- & \underline{3x + 15} \\
 & 0
\end{array}
$$

$$\therefore \frac{6x^3 + 28x^2 - 7x + 15}{x + 5} = 6x^2 - 2x + 3$$

Example 2: Divide $-5x^3 - 27x^2 + 23x + 30$ *by* $(x + 6)$

$$
\begin{array}{rl}
 & -5x^2 + 3x + 5 \\
= \quad (x+6) & \overline{)\,-5x^3 - 27x^2 + 23x + 30} \\
- & \underline{-5x^3 - 30x^2} \\
 & 3x^2 + 23x \\
- & \underline{3x^2 + 18x} \\
 & 5x + 30 \\
- & \underline{5x + 30} \\
 & 0
\end{array}
$$

$$\therefore \frac{-5x^3 - 27x^2 + 23x + 30}{x + 6} = -5x^2 + 3x + 5$$

Finding the Remainder

After a division of a polynomial, if there is anything left then it is called the remainder. The answer is called the quotient. If there is no remainder then what you are dividing by is said to be a factor.

Example 1: Divide $2x^3 - 5x^2 - 16x + 10$ by $(x - 4)$ and state the remainder and quotient

$$\begin{array}{r} 2x^2 + 3x - 4 \\ = (x-4)\overline{)2x^3 - 5x^2 - 16x + 10} \\ -\ \underline{2x^3 - 8x^2} \\ 3x^2 - 16x \\ -\ \underline{3x^2 - 12x} \\ -4x + 10 \\ -\ \underline{-4x + 16} \\ -6 \end{array}$$

$\therefore$ The remainder is -6 and the quotient is $2x^2 + 3x - 4$

Example 2: Divide $2x^3 + 9x^2 + 25$ by $(x + 5)$ and state the remainder and quotient

$$\begin{array}{r} 2x^2 - x + 5 \\ = (x+5)\overline{)2x^3 + 9x^2 + 0x + 25} \\ -\ \underline{2x^3 + 10x^2} \\ -x^2 + 0x \\ -\ \underline{-x^2 - 5x} \\ 5x + 25 \\ -\ \underline{5x + 25} \\ 0 \end{array}$$

Remember to use 0x as there is no x term.

$\therefore$ The remainder is 0 and the quotient is $2x^2 - x + 5$

Factor Theorem

To find out the factors of a polynomial you can quickly substitute in values of x to see which give you a value of zero.

Example 1: Given $f(x) = x^3 + x^2 - 4x - 4$. Use the factor theorem to find a factor

$$f(x) = x^3 + x^2 - 4x - 4$$

$$f(1) = (1)^3 + (1)^2 - (4 \times 1) - 4$$

$$= -6 \qquad \therefore (x - 1) \text{ is not a factor}$$

$$f(2) = (2)^3 + (2)^2 - (4 \times 2) - 4$$

$$= 0 \qquad \therefore (x - 2) \text{ is a factor}$$

To go from the x value to the factor simply put it into a bracket and change the sign.

Remember this:-

$(x + 3)(x - 2) = 0$

$x = -3$ and $x = 2$

We are now going in reverse by putting the x value back into the brackets

Example 2: Given $f(x) = 3x^2 + 8x^2 + 3x - 2$. Factorise fully the given function

$$f(x) = 3x^3 + 8x^2 + 3x - 2$$

$$f(1) = 3 \times (1)^3 + 8 \times (1)^2 + (3 \times 1) - 2$$

$$= 12 \qquad \therefore (x - 1) \text{ is not a factor}$$

$$f(2) = 3 \times (2)^3 + 8 \times (2)^2 + (3 \times 2) - 2$$

$$= 60 \qquad \therefore (x - 2) \text{ is not a factor}$$

$$f(-1) = 3 \times (-1)^3 + 8 \times (-1)^2 + (3 \times -1) - 2$$

$$= 0 \qquad \therefore (x + 1) \text{ is a factor}$$

To fully factorise it we now need to find the quotient. We do this by dividing by the factor we have just found.

$$
\begin{array}{rl}
 & \;\; 3x^2 + 5x - 2 \\
= \ (x+1) & \overline{)\,3x^3 + 8x^2 + 3x - 2} \\
 & -\ \underline{3x^3 + 3x^2} \\
 & \qquad\quad 5x^2 + 3x \\
 & \qquad -\ \underline{5x^2 + 5x} \\
 & \qquad\qquad\quad -2x - 2 \\
 & \qquad\qquad -\ \underline{-2x - 2} \\
 & \qquad\qquad\qquad\qquad 0
\end{array}
$$

$$\therefore (x+1)(3x^2 + 5x - 2) = (x+1)(3x-1)(x+2)$$

We have factorised the quotient further to get the fully factorised answer

Using the Factor Theorem to find the Remainder

If the polynomial $f(x)$ is divided by $(ax - b)$ then the remainder is $f\left(\frac{b}{a}\right)$

Example 1: Find the remainder when $f(x) = 4x^4 - 4x^2 + 8x - 1$ is divided by $(2x - 1)$

Using $(ax - b)$ then a = 2, b = 1 so $f\left(\frac{1}{2}\right)$

$$f(x) = 4x^4 - 4x^2 + 8x - 1$$

$$f\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^4 - 4 \times \left(\frac{1}{2}\right)^2 + \left(8 \times \frac{1}{2}\right) - 1$$

$$= \frac{4}{16} - \frac{4}{4} + \frac{8}{2} - 1$$

$$= \frac{1}{4} - 1 + 4 - 1$$

$$= 2\frac{1}{4}$$

$\therefore$ Remainder is $2\frac{1}{4}$

Example 2: Find the remainder if $f(x) = x^3 - 20x + 3$ is divided by $(x - 4)$

Using $(ax - b)$ then a = 1, b = 4 so $f(4)$

$$f(x) = x^3 - 20x + 3$$

$$f(4) = (4)^3 - (20 \times 4) + 3$$

$$= 64 - 80 + 3$$

$$= -13$$

$\therefore$ Remainder is -13