

Lesson 6: Factorising quadratics: An investigation.

Aim: 'I will be able to investigate through an open-ended problem, make a prediction and prove it.'

This investigation brings together collecting like terms, factorising and expanding quadratic equations. The best students can move into proof using symbolic form and manipulation to prove their result.

Prior Knowledge: Students should be able to factorise expressions containing a squared term. **You can use the worksheet ;Investigating Factorising Quadratics.'**

Length

This will take more than one lesson; ideally spend one lesson setting up the problem and one lesson investigating. A further lesson or homework can be spent writing up the problem.

Discussion 1 (10 minutes)

Students can discuss these answers with partners or in groups.

Arrange some of the following numbers into the boxes to make factorised quadratics.

6	5	3	2	9	1	4	15	-3	2	3	5
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$$(x + \square)(x + \square) = x^2 + \square x + 9$$

$$(x + \square)(x - \square) = x^2 + \square x - 15$$

$$(x + \square)(\square x + \square) = 2x^2 + \square x + 10$$

Extension:

$$6x^2 - 7x + 2$$

from looking at the equation, how do you know that the factors will not work?

This activity focuses on students thinking of numbers that multiply to give the constant term and collect to give the linear term. Although the gap-filling exercise seems simple this stimulates and encourages the discussion of how the two numbers in the brackets give rise to the linear term.

Discussion 2 (10 minutes)

This can be teacher led, or in partners/groups.

think through and write answers in complete sentences.

Question 1 $6x^2 - 7x + 2$ from looking at the equation, how do you know that the factors will not work?

Question 2 $2x^2 + 11x + 5$ from looking at the equation, how do you know that the factors will not work?

Question 3 **hard** $2x^2$ $+ 7$

Question 3 **easier** x^2 $+ 7$

Choose from the terms below, which could be the middle term for each of the quadratics?

+9x	+7x	-9x	+2x
-15x	+15x	-x	+14x
-14x	+8x	-8x	-7x

Students are again encouraged to think how the factors of the constant term give rise to the linear term. Question 3 indicates that whilst the squared and constant terms are independent the linear term is dependent – yet there are several options.

Setting up the investigation (10 - 15 minutes)

$(x + \square)(\square x + \square)$	start	end				
	1	2	3	4	5	6
★ $(x + 1)(2x + 3)$						$2x^2 + 5x + 3$
★ $(x + 1)(3x + 2)$						$3x^2 + 5x + 2$
★ $(x + 2)(1x + 3)$						$x^2 + 5x + 6$
★ $(x + 2)(3x + 1)$						$3x^2 + 7x + 2$
★ $(x + 3)(1x + 2)$						$x^2 + 5x + 6$
★ $(x + 3)(2x + 1)$						$2x^2 + 7x + 3$

Students pick three numbers. These are then placed in the boxes in every possible order to generate all **six** quadratics.

Students expand these, sum all 6 expressions in to 1 quadratic, collect like terms, and then factorise. When comparing their solutions with their peers students will see that $x + 1$ is always a factor. The task can be differentiated by introducing negative numbers. It can also be extended further by using a, b and c as the numbers, to generate an algebraic proof of the result.