Work, Energy and Power

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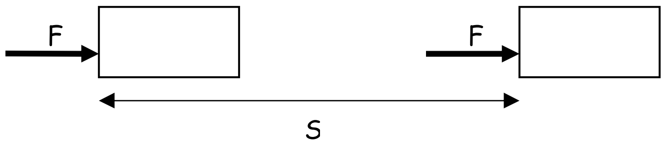
Work

Work done by a constant force



If the point of application of a force of F Newtons moves through a distance *s* metres in the direction of the force then the **work done by the force** is given by:

Work = F x s

The unit of work is the joule (J)

<u>Example 1</u>

The figure shows a box which is pulled at a constant speed across a horizontal surface by a horizontal rope. When the box has moved a distance of 9m the work done is 54 J. Find the constant resistance to the motion.

Work = force × distance moved

54 = force × 9

Resistance to motion = 6N

Work done against gravity

To raise a particle of mass *m* kg vertically at a constant speed you need to apply a force of *mg* Newtons vertically upwards. If the particle is raised a distance of *h* metres the work done against gravity is *mgh* joules. Work is done against gravity when the particle is moving either vertically or at an angle to the horizontal, for example on an inclined plane, but not when the particle is moving along a horizontal plane.

<u>Example 2</u>
Find the work done by a child of mass 16kg whilst climbing to the top of a slide of vertical height 9m.

Work done against gravity = mgh

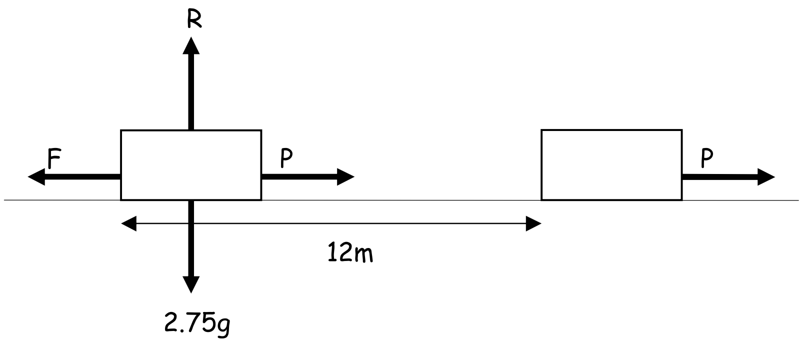
$= 16 \times 9.8 \times 9$

= 1410J

Work Done Against Friction

Example 3

A horizontal force P pulls a body of mass 2.75kg a distance of 12m across a rough horizontal surface, coefficient of friction 0.2. The body moves with a constant velocity and the only resisting force is that due to friction. Find the work done against friction.



Resolving perpendicular to the plane:

$$R = 2.75g$$

Using $F = \mu R$

$$F = 0.25 \times 9.8 \times 2.75$$

$$F = 6.74N$$

Therefore:

Work = force × distance moved

$= 6.74 \times 12$

$= 80.9J$

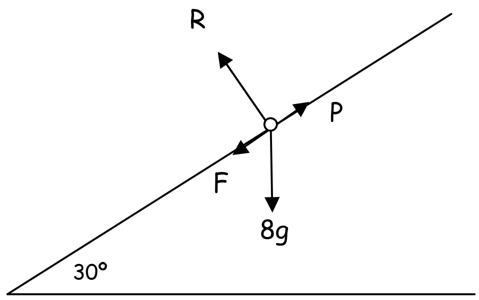
Exam questions will involve work done against gravity and friction.

Example 4

A particle of mass 8 kg is pulled at constant speed a distance of 28m up a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the particle and the surface is 0.2. Assuming the particle moves up a line of greatest slope, find:

(a) the work done against friction.

(b) the work done against gravity.



(a) Work done against friction = force × distance moved

Resolving perpendicular to the plane gives:

$R = 8g \cos 30°$

Using $F = \mu R$ $\quad F = 0.2 \times 8g \cos 30°$

$F = 13.58N$

Work done against friction = 13.58 × 28
= 380J

(b) Work done against gravity = mgh

Remember that we need to find the vertical displacement

Work done against gravity = 8 × g × 28 sin30°
=1097.6
=1100J

Questions 1

1 A man building a wall lifts 75 bricks through a vertical distance of 3.5m. If each brick weighs 5kg, how much work does the man do against gravity?

2 Find the work done against gravity when a person of mass 90kg climbs a vertical distance of 32m.

3 A box of mass 12kg is pulled a distance of 25m across a horizontal surface against resistances totaling 50N. If the body moves with uniform velocity, find the work done against the resistances.

4 A horizontal force pulls a body of mass 6kg a distance of 11m across a rough horizontal surface, coefficient of friction 0.25. The body moves with uniform velocity and the only resisting force is that due to friction. Find the work done.

5 A horizontal force pulls a body of mass 2.5kg a distance of 25m across a rough horizontal surface, coefficient of friction 0.3. The body moves with uniform velocity and the only resisting force is that due to friction. Find the work done.

6 A smooth surface is inclined at an angle of 30° to the horizontal. A parcel of mass 18kg lies on the surface and is pulled at a uniform speed a distance of 7.5m up a line of greatest slope. Find the work done against gravity.

7 A surface is inclined at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. A body of mass 78kg lies on the surface and is pulled at a uniform speed a distance of 7.5m up a line of greatest slope against resistances totaling 60N. Find:

a) the work done against gravity.
b) the work done against the resistances.

8 A rough surface is inclined at an angle $\cos^{-1}\left(\frac{12}{13}\right)$ to the horizontal. A body of mass 140kg lies on the surface and is pulled at a uniform speed a distance of 45m up the surface by a force acting along the line of greatest slope. The coefficient of friction between the body and the surface is $\left(\frac{2}{7}\right)$ Find:

a) the frictional force acting.
b) the work done against friction.
c) the work done against gravity.

9 A rough surface is inclined at an angle $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ to the horizontal. A body of mass 90kg lies on the surface and is pulled at a uniform speed a distance of 15m up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\left(\frac{1}{8}\right)$ Find:

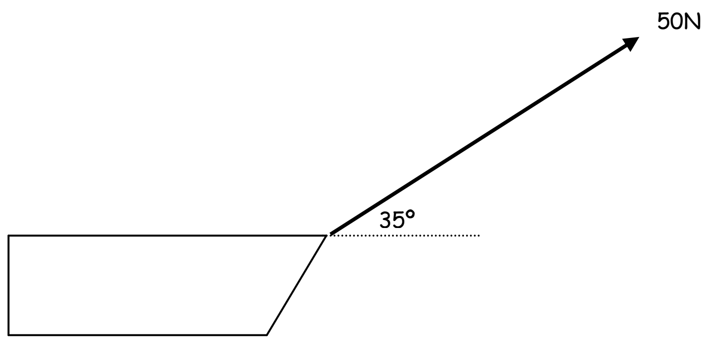
a) the work done against friction.
b) the work done against gravity.

Forces at an angle to the direction of motion

Consider a particle P resting on a horizontal surface. If a force of magnitude F inclined at an angle θ to the horizontal causes the particle P to move along the surface while remaining in contact with the surface, you can resolve the force into its horizontal and vertical components.

For a force at an angle to the direction of motion:
Work done = component of force in direction of motion x distance moved in the same direction.

Example 5
A sledge is pulled across a smooth horizontal floor by a force of magnitude 50 N inclined at 35° to the horizontal. Find the work done by the force in moving the packing case a distance of 23m.



The horizontal distance is 23m so we must consider the horizontal component of the force.

Horizontal component = 50 × cos35° = 40.96N

Work done = force × distance moved

= 40.96 × 23 = 942J

Energy

The energy of a body is a measure of the capacity which the body has to do work. When a force does work on a body it changes the energy of the body. Energy exists in a number of forms, but we will consider two main types: kinetic energy and potential energy.

Kinetic energy

The kinetic energy of a body is the energy that it possesses by virtue of its motion. When a force acts on a body to increase its speed, then the work done equates to the increase in kinetic energy of the body (provided that no other forces are involved).

If a constant force F acts on a body of mass m, which is initially at rest on a smooth horizontal surface, then after a distance s the body has velocity v. So by considering the work equation:

Work done against friction = force × distance moved

$$= F \times s$$

Using the fact that $F = ma$

and from the constant acceleration equations

$$v^2 = u^2 + 2as$$

$$u = 0$$

$$a = \frac{v^2}{2s}$$

Therefore: $$\text{work done} = m \times \frac{v^2}{2s} \times s$$

$$= \frac{mv^2}{2}$$

Therefore $\frac{mv^2}{2}$ is said to be the kinetic energy of a mass m moving with velocity v.

Example 6

A particle of mass 0.25kg is moving with a speed of $7ms^{-1}$. Find its kinetic energy.

Using the formula:

$$KE = \frac{1}{2}mv^2:$$

$$KE = \frac{1}{2} \times 0.25 \times 7^2$$

$$KE = 6.13J$$

Example 7

A particle of mass 4 kg is being pulled across a smooth horizontal surface by a horizontal force. The force does 46 J of work in increasing the particle's velocity from $3ms^{-1}$ to pms^{-1}. Find the value of p.

The change in Kinetic Energy is given by the formula:

$$\Delta KE = \frac{1}{2}m(v^2 - u^2)$$

Where u is the initial and v is the final, velocity respectively. Since there are no other forces involved the change in Kinetic Energy must equal the work done by the force.

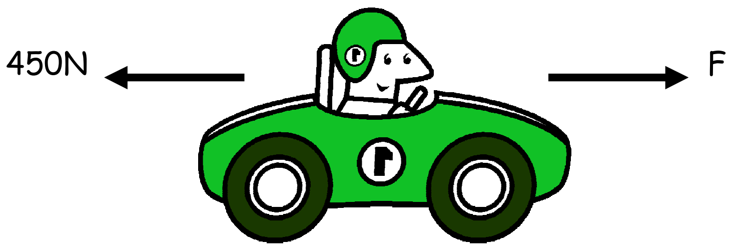
$$46 = \frac{1}{2} \times 4(v^2 - 3^2)$$

$$23 = v^2 - 3^2$$

$$v = \sqrt{32} = 4\sqrt{2}$$

Example 8

A van of mass 1600 kg starts from rest at a set of traffic lights. After travelling 240 m its speed is 23ms^{-1} . Given that the car is subject to a constant resistance of 450 N find the constant driving force.



Initially the Kinetic Energy is zero.
Finally the Kinetic Energy is:

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1600 \times 23^2$$

$$= 423200\text{J}$$

Work done against the resistance = force × distance moved

= (F - 450) × 240

The change in Kinetic Energy equates to the work done. Therefore:

$$423200 = (F - 450) \times 240$$

$$F = 1460\text{N}$$

Potential energy

If the particle is raised a distance of *h* metres the work done against gravity is *mgh* joules. The work done against gravity equates to the increase in potential energy. If the particle is lowered then the potential energy decreases. When working with potential energy questions it is vital that a zero potential energy point is decided upon.

Remember: P.E. = *mgh*

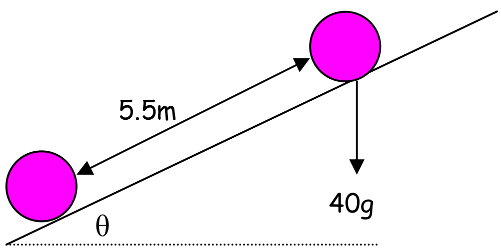
Example 9

A child of mass 14 kg is raised vertically through a distance of 1.8 m. Find the increase in potential energy.

$$\text{P.E.} = mgh$$
$$= 14 \times 9.8 \times 1.8$$
$$= 247\text{J}$$

Example 10

A child of mass 40kg slides 5.5m down a playground slide inclined at an angle of $\arcsin\left[\frac{4}{7}\right]$ to the horizontal. Model the child as a particle and the slide as an inclined plane and hence calculate the potential energy lost by the child.



The important thing to remember is that potential energy changes after a change in height. The 5.5m displacement is **not** the value of h.

$\arcsin\left[\frac{4}{7}\right]$ means that $\sin\theta = \left[\frac{4}{7}\right]$.

$$h = 5.5 \times \sin\theta$$

$$h = 5.5 \times \left[\frac{4}{7}\right] = 3.142\text{m}$$

Therefore:

$$PE = mgh$$

$$= 40 \times 9.8 \times 3.142$$

$$= 1230\text{J}$$

Questions 2

1 A body of mass 7.5kg, initially moving with velocity $4\,\text{ms}^{-1}$, increases its kinetic energy by 55J. Find the final speed of the body.

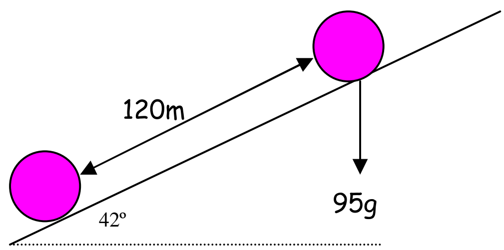
2 Find the increase in kinetic energy when a stationary van of mass 1400kg accelerates at $4\,\text{ms}^{-2}$ for 6 seconds.

3 Find the loss in kinetic energy when a car decelerates from $45\,\text{kmh}^{-1}$ to rest (take care with units).

4 Find the potential energy lost by a lift of mass 750kg as it descends 65m.

5 A body of mass 14kg, initially moving with a speed of $18\,\text{ms}^{-1}$, experiences a constant retarding force of 15N for 5 seconds. Find the kinetic energy of the body at the end of this time.

6 A ski jumper of mass 95kg sets off from the top of the run and travels a distance of 120. If the run is inclined at an angle of 42° to the horizontal find the loss in potential energy.



Conservation of Energy

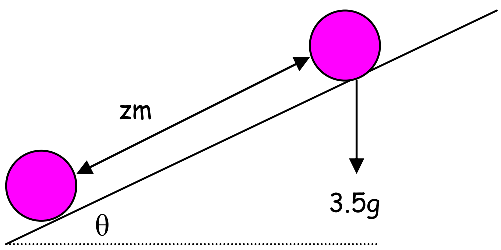
Consider the situation of a moving body where no work is done against friction and that gravity is the only other force present then:

Total Energy = Kinetic Energy + Potential Energy = Constant

Or in other terms:

Total Initial Energy = Total Final Energy

<u>Example 11</u>
A particle of mass 3.5kg is released from rest and slides down a smooth plane inclined at $\arcsin\left(\frac{4}{7}\right)$ to the horizontal. Find the distance travelled while the particle increases its velocity to $5ms^{-1}$. Let the distance travelled be z m.



Always consider PE and KE at the start and finish.

Initial KE = 0

$$\text{Final KE} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 3.5 \times 5^2$$

$$= 43.75\text{J}$$

Let the final position be the zero potential energy point.

Initial PE = mgh

$$= 3.5 \times 9.8 \times z\sin\theta$$
$$= 34.3 \times z \times \left(\frac{4}{7}\right)$$

By the Conservation of Energy Principle

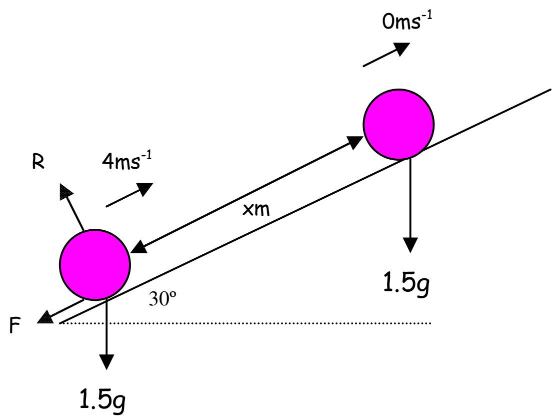
Initial (KE + PE) = Final (KE + PE)

$$0 + 34.3 \times z \times \left(\frac{4}{7}\right) = 43.75 + 0$$

$$z = 2.23\text{m}$$

<u>Example 12</u>

A ball of mass 1.5kg is projected up a rough plane inclined at an angle of 30° to the horizontal with a speed of 4 ms^{-1}. Given that the coefficient of friction between the particle and the plane is 0.2 find the distance the particle moves up the plane before coming to rest.



Let the initial position be the zero PE point.

$$\text{Initial KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.5 \times 4^2$$

$$\text{Initial KE} = 12\text{J}$$

Final KE = 0

Final PE = mgh

$= 1.5 \times 9.8 \times x\sin 30°$

$= 7.35x\text{J}$

The difference between the initial energy and the final energy is the work done against friction. Therefore:

Initial (KE + PE) - Final (KE + PE) = Work Done **(1)**

To calculate the friction we first need to find the normal reaction force.

Resolving perpendicular to the plane gives:

$R = 1.5g\cos 30°$

$R = 12.73\text{N}$

Using $F = \mu R$ $F = 12.73 \times 0.2 = 2.55\text{N}$

Work done = force × distance moved

$= 2.55 \times x$

Substituting all of the values into equation **(1)** gives:

$$(12 + 0) - (7.35x) = 2.55x$$

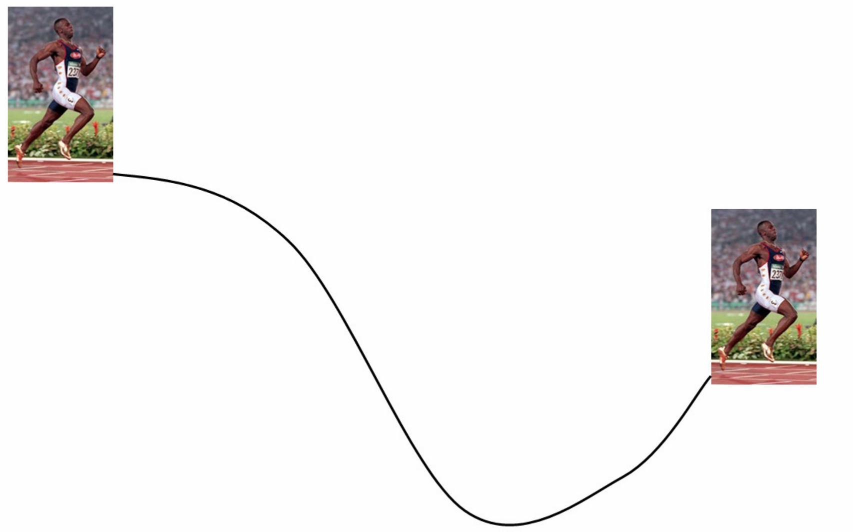
$$12 = 9.9x$$

$$x = 1.21m$$

The above type of question appears regularly on M2 papers and it is usual to be told that the question must be solved by application of the energy principle. Otherwise constant acceleration equations could be used.

Exam Type Question

Michael Johnson reaches the top of a hill with a speed of $6.5ms^{-1}$. He descends 50 m and then ascends 28 m to the top of the next incline. His speed is now $4.5ms^{-1}$. Michael has a mass of 83 kg. The total distance that Michael runs is 400m, and there is a constant resistance to motion of 9 N. By consideration of the energy principle find the work done by Michael.



Start by considering the KE and PE at the beginning of the race. Assume that the zero PE point is at the end. Therefore we only need to consider a 22m vertical displacement when working out the change in the PE.

$$\text{Initial KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 83 \times 6.5^2$$

$$\text{Initial KE} = 1753J$$

Initial PE = mgh

= 83 × 9.8 × 22

= 17895J

Total Energy initially = 19648J

$$\text{Final KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 83 \times 4.5^2$$

$$\text{Final KE} = 840\text{J}$$

Final PE = 0

Total Energy Finally = 840J

Therefore the change in energy = 18808J
This change in energy has obviously been brought about by work. The work done against the resistance is:

Work done against resistance = 9 × 400 = 3600J

The difference between the change in energy and the work done against resistances must be the work done by Michael

Work done by Michael = 18808 – 3600

= 15208J

Power

Power is a measure of the rate at which work is being done. The unit of work is the Watt. If 1 Joule of work is done in 1 second then the rate of work is 1 Watt.

Power = Force × Distance moved in one second

or

Power = Force × Velocity

Example 13

A force of magnitude 750 N pulls a car up a slope at a constant speed of $9ms^{-1}$. Given that the force acts parallel to the direction of motion find, in kW, the power developed.

Power = Force × Velocity

= 750 × 9

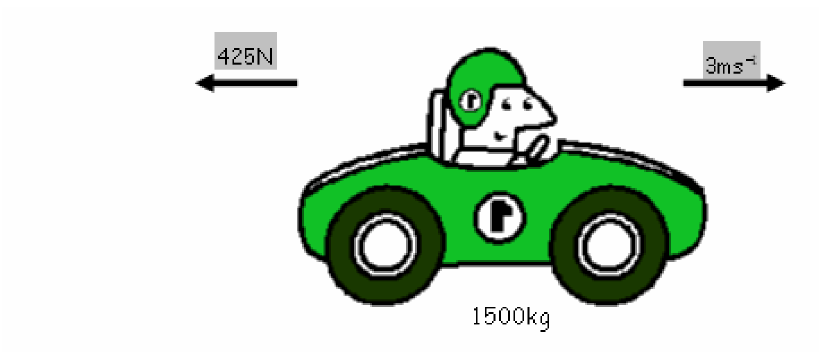
= 6750 = 6.75kW

Example 14

A car of mass 1500kg is travelling along a level road against a constant resistance of magnitude 425 N. The engine of the car is working at 6 kW. Calculate:

(a) the acceleration when the car is travelling at $3ms^{-1}$

(b) the maximum speed of the car.



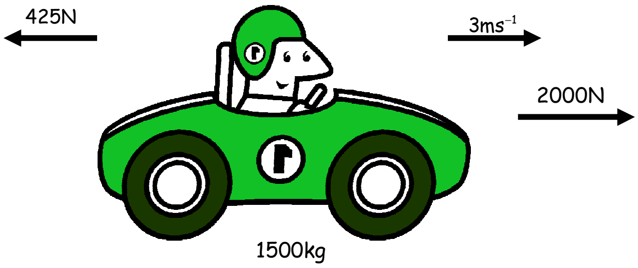
(a) Using the power equation to find the **engine** force

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$6000 = \text{Force} \times 3\text{ms}^{-1}$$

$$\text{Force} = 2000\text{N}$$

Applying the engine force to the diagram:



Using F = ma

$$2000 - 425 = 1500a$$

$$a = 1.05\text{ms}^{-2}$$

(b) The maximum speed occurs when the acceleration is zero. At this point the engine force equates to the resistance. By using the power equation again:

$$\text{Power} = \text{Force} \times \text{Velocity}$$

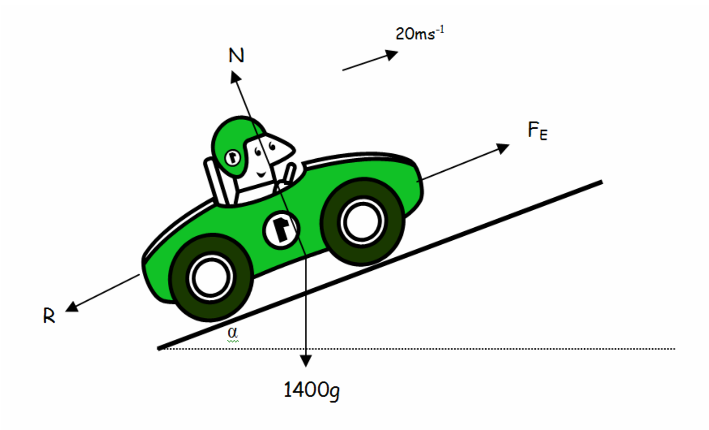
$$6000 = 425 \times v$$

$$v = 14.1\text{ms}^{-1}$$

<u>Example 15</u>

A car of mass 1400kg is moving up a hill of slope $\arcsin\left(\frac{1}{12}\right)$ at a constant speed of 25ms^{-1}. If the power developed by the engine is 32 kW find the resistance to motion.

At the top of the hill the road becomes horizontal. Find the initial acceleration, assuming the resistance to be unchanged.



If the engine is working at a rate of 32kW the engine force F_E can be found

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$32000 = F_E \times 25$$

$$F_E = 1280\text{N}$$

Since the velocity is constant the weight component of the car acting downhill and the resistance to motion must equate to the engine force.

Therefore:

$$1400g \sin\alpha + R = 1280$$

$\sin\alpha = \left(\frac{1}{12}\right)$

$$1143.33 + R = 1280$$

$$R = 137\text{N}$$

As the vehicle reaches the top of the hill only R is acting to slow the car down. With the engine force still being 1280N we can use F = ma.

$$1280 - 137 = 1400 \times a$$

$$a = 0.816\text{ms}^{-2}$$

Questions 3

1 An athlete is running on a level track at a constant speed of $9ms^{-1}$. If the resistances to motion equal 45N find the rate at which the athlete is working.

2 A motorcycle is driven along a horizontal road against a constant resistance of 350N. Find the maximum speed achieved by the motorcycle if the engine works at a rate of
a) 3kW b) 5kW c) 9kW

3 A van traveling at maximum speed travels along a horizontal road. If the engine is working at a rate of 16kW and the speed attained is $42ms^{-1}$ calculate the magnitude of the resistances.

4 A cyclist and bike have a combined mass of 85kg. If the cyclist is working at a constant rate of 250W against resistances totaling 18N calculate the maximum speed.
The cyclist then ascends a slope of incline 10°. If the cyclist continues to work at the same rate and the resistances remain unchanged calculate the maximum speed up the incline.

5 A small van travels along a level road against constant resistance to motion of 450N. The mass of the van is 1200kg and its maximum speed is $35ms^{-1}$. Calculate the maximum speed of the same van on an incline of $\arcsin\left(\frac{1}{35}\right)$ assuming that the resistance and the rate at which the engine works remain unchanged.

6 Bob is busy removing roof tiles from a house. He sends the tiles down a chute of length 12m, inclined at an angle of 35° to the horizontal. The roof tiles weigh 4.5kg each and are released from rest at the top of the chute. When the tile reaches the bottom it has a speed of $6.5ms^{-1}$. Calculate:

a) the potential energy lost by the tile.

b) the frictional force, assumed constant, which acts on the tile.

c) the coefficient of friction between the tile and the chute.

If Wendy pushes a tile from the top of the chute with a speed of $1.5ms^{-1}$ what is its speed at the bottom?

7 Thomas the engine has a mass of 7.5×10^5kg and whilst traveling along a horizontal track he experiences resistance to motion of magnitude 1.75×10^5N.

a) Calculate the force that Thomas produces when the acceleration is $1.5ms^{-2}$.

b) Calculate the power produced at the instant that Thomas is traveling at $12ms^{-1}$.

c) If Thomas then works at a constant rate of 600kW, find his greatest possible speed along the track.

8 A Milk float of mass 1500kg moves with a constant speed of $7.5ms^{-1}$ up a slope with incline $\arcsin\left(\frac{1}{8}\right)$. Given that the engine is working at a rate of 21kW, find the resistance to motion.

9 With its engine working at a constant rate of 10kW, a car of mass 750kg can descend a slope of incline 1 in 50 at twice the steady speed that it can ascend the same slope with the resistance to motion remaining constant. Find the resistance to motion and the speed of ascent.

10 A car of mass 700kg is moving along a straight horizontal road against a constant resistive force of magnitude 450N. The engine of the car is working at a rate of 8.76kW.

a) find the acceleration of the car at the instant when its speed is $12ms^{-1}$.

The car now moves up a straight road inclined at an angle of

$\arcsin\left(\frac{1}{20}\right)$ to the horizontal against the same resistive force of magnitude 450N. If the car moves at a constant speed of $V\text{ms}^{-1}$ whilst working at a rate of 12kW, find the value of V.

11 A lorry of mass 1500 kg moves along a straight horizontal road. The resistance to the motion of the lorry has magnitude 750 N and the lorry's engine is working at a rate of 36 kW.

(*a*) Find the acceleration of the lorry when its speed is 20 ms^{-1}.

The lorry comes to a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{10}$. The magnitude of the resistance to motion from non-gravitational forces remains 750 N.

The lorry moves up the hill at a constant speed of 20 ms^{-1}.

(*b*) Find the rate at which the lorry's engine is now working.